Missing the Wealthy in the HFCS: Micro Problems with Macro Implications

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Abstract

Macroeconomic aggregates on households' wealth have a long tradition and are widely used to analyse and compare economies, yet they do not provide any information about the distribution of assets and liabilities within the population. The *Household Finance and Consumption Survey* (HFCS) constitutes a rich source of micro data that can be used to link macro aggregates with distributional information to compile *Distributional National Accounts* for wealth. Computing aggregates from this survey usually yields much lower amounts than reported by macroeconomic statistics. An important source of this gap may be the lack of the wealthiest households in the HFCS which heavily contribute to total wealth. This paper combines a semi-parametric Pareto model estimated from survey data and observations from rich lists with a stratification approach making use of HFCS portfolio structures to quantify the impact of missing wealthy households on instrument-specific aggregates. It proposes an analytical as well as a simulation version of the methodology. We analyse data for Austria and Germany, and find that adjusting for the *missing wealthy* increases instrument-specific aggregates, pushes up inequality even further, has large effects on equity, but explains less than ten percentage points of the micro-macro gap for most other instruments. *JEL codes:* D31, E01

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1 Non-technical summary

Analysing, assessing and comparing economies is usually done by means of macroeconomic aggregates or indicators derived therefrom. These indicators, however, do not tell anything about the distribution of income, consumption, saving and wealth within the population although distributional information is needed to thoroughly design and assess the impact of policies (including monetary policy), for research purposes, as a source of information for the greater public and, generally, to assess an economy in a comprehensive way.

The need of distributional information for the household sector was also pointed out in the *Stiglitz-Sen-Fitoussi report* and by the *G20 Data Gaps Initiative*.[1] The joint *OECD-Eurostat Expert Group on Measuring Disparities in a National Accounts Framework* focuses on distributional indicators for income and consumption, and the *ECB Expert Group on Linking Macro and Micro Data for the Household Sector* (EG-LMM) works on linking micro data obtained from the *Household Finance and Consumption Survey* (HFCS) with financial accounts data to derive distributional indicators for wealth. The article at hand emerged in the course of the work for the EG-LMM.

All these initiatives should eventually lead to complementing and breaking-down national accounts by homogeneous household groups such as wealth or income quintiles, or types of households. These *Distributional National Accounts* (DINA) should ideally be compiled in a standardised and harmonised way. Distributional figures also need to be integrated within the national accounts framework, which means that eventually distributional split-ups must sum up to national account aggregates. This integration is needed to avoid confusions among users, and enable a consistent and comprehensive discussion about distributional phenomena such as wealth inequality.

For the purpose of integration the comparability of the definitions of instruments in national accounts and the micro source used needs to be assessed and should be aligned to the extent possible. The EG-LMM has worked out a bridging table that classifies HFCS instruments as of high, medium and low conceptual comparability with the financial accounts (EG-LMM, 2017). In this article we focus on the highly comparable instruments loans (liabilities), deposits (sight and saving accounts), bonds (debt securities) and mutual funds. Additionally, we perform some calculations for equity (medium comparability) and real estate (not yet assessed).

When calculating aggregates from the HFCS, these aggregates are usually much lower than national accounts figures for highly comparable instruments. A current focus therefore lies on investigating the reasons causing this macro-micro gap. In general, there are two types of error sources: errors influencing the shape of the distribution and randomly spread errors impacting the total but not the distribution. Whereas the latter does not raise concerns, the former is crucial when aiming for distributional figures. There are many reasons why we would expect differences in aggregates and one of them are what is called here the *missing wealthy*.

The HFCS, just like any other wealth-related survey, does not perfectly reflect the very top of the wealth distribution, as the participation of very wealthy households in such surveys is less likely. Due to the high concentration of wealth at the very top, particularly good information from this part of the distribution however would be needed. The missing wealthy are clearly an error source influencing the shape of the distribution and therefore assessing this issue constitutes an important step towards distributional figures.

Thus, the aim of this article is twofold: First, we quantify the contribution of the missing wealthy toward the macro-micro gap, i.e., we measure the increase in coverage ratios (defined

[1] See section 2 for further details and references.

as HFCS aggregates over national accounts aggregates) after adjusting for the missing wealthy. Second, we show how such quantification can and should be used in the compilation of distributional national account figures. We perform a case study for Austria and Germany.

As the HFCS lacks observations from the very top, we substitute the top tail of the empirical wealth distribution implied by the HFCS by a theoretical model, namely the Pareto distribution. In the second step, we break down the Pareto-inflated wealth distribution on instruments.

We estimate the parameter of the Pareto distribution by using HFCS observations with a net worth of at least 1 million EUR together with additional data listing the wealth of the richest persons and families in a country. These so-called rich lists are compiled by newspapers, and heavily rely on assumptions and estimation, which is why these data points only enter the calculations indirectly. Robustness checks show that overall results do not strongly depend on the exact amounts reported by rich lists. Still, to decrease the dependency on such less trustworthy data sources and increase the quality of distributional national accounts figures, collecting more and making available existing administrative wealth-related data for statistical purposes would be needed.

In this article, we propose two complementing approaches to break down the adjusted wealth distributions on instruments: an analytical and a simulation approach. While the analytical approach is fast, easy to implement and well suited to calculate adjusted instrument-specific aggregates, the simulation approach additionally provides distributional information about the top tail as well as variation measures used for confidence intervals or there like. For this breakdown we rely on portfolio structures observed in the HFCS and propose to model portfolios at the very top of the distribution by using top observations from any HFCS country as we assume that investment behaviour of the very rich does not vary strongly across EU countries.

Against widely spread prior believes the missing wealthy do not explain large parts of the macro-micro gap for highly comparable instruments. For most instruments, less than 10 percentage points of this gap can be attributed to the missing wealthy still leaving significant parts (ranging between 42 and 88 percentage points) unexplained. This suggests that there is not *that one single source* explaining the observed under-coverage for these instruments but rather hints toward a longer list of small – but in sum important – reasons.

In contrast to liabilities, bonds, deposits and mutual funds, we find that micro data based aggregates for equity (and to a lesser extend also real estate) increase heavily when adjusting for the missing wealthy. Also without this adjustment, HFCS totals for equity usually exceed what is observed in financial accounts. Our methodology pushes up these aggregates even further. The large increase in equity is not surprising as the wealthiest of the wealthy hold great parts of their total wealth in the form of company shares or other forms of equity. Other financial instruments such as mutual funds, bonds or deposits are much less important (in relative terms) for this part of the population.

While a large jump in total equity is clearly expected and any other result would decrease confidence in our methodology, this rise however also points toward a fundamental problem of the macro-micro linkage: the poor conceptual comparability of equity as defined in the financial accounts and the HFCS definition of business wealth. As equity forms a large and important part of total wealth better aligning definitions must therefore be a top priority for future work.

Once the impact of the missing wealthy is quantified, we can use these results to enhance distributional national accounts figures. The way how we make use of such quantification in the compilation process of distributional figures is not limited to the impact of the missing wealthy, but can be applied whenever a quantification of an error source affecting the shape of the distribution is possible.

In general, whenever there is a gap between micro and macro aggregate and the assumption is justified that the gap is a consequence of errors affecting all parts of the distribution equally, distributional figures obtained from micro sources are scaled up (or down) to achieve consistency with macro aggregates. In the case of under-coverage, this is done by proportionally distributing the "gap euros" over all groups by which the total is split up (e.g., net worth or income quintiles) whereas in case of over-coverage, excess euros are proportionally withdrawn from all groups. Such a procedure guarantees that the distributional relationships between groups are conserved.

If an error source has diverse effects across the distribution and these effects can be quantified, this information can be used before scaling: By quantifying the impact of the missing wealthy as done in this article, we know that actually a larger proportion of the aggregate should be allocated to the top net worth group than implied by the HFCS. In the more common case of under-coverage, this has two effects: First, the remaining gap that needs to be spread over the entire distribution is smaller, and second, the distribution is more skewed than before thus the top net worth quintile receives a larger share when proportionally distributing the remaining gap. Both effects push up the amount attributed to the top net worth group.

The methodology we propose in this article can thus be used in two ways: First, as a test to assess whether the *missing wealthy* constitute a serious problem for a particular instrument, country and point in time, and, second, to make adjustments to distributional figures whenever needed.

2 Introduction

While general macroeconomic statistics such as national accounts show the evolution of overall household wealth and income, they are unable to reveal information on the distribution within an economy, and on how the distribution evolves over time.

In recent years, there has been renewed interest in wealth distributions – and in particular in wealth inequality – among the greater public as well as academics, politicians, and policy makers. Thomas Piketty's *Capital in the Twenty-First Century* has contributed largely to these flourishing discussions (Piketty, 2014). Piketty and Zucman (2014) find increasing wealth-to-income ratios for the top eight developed economies in recent decades and conclude that because of this

> [...] the inequality of wealth, and potentially the inequality of inherited wealth, is likely to play a bigger role for the overall structure of inequality in the twenty-first century than it did in the postwar period. (Piketty and Zucman, 2014, page 1261)

The availability of high-quality data on wealth distributions constitues the first and most important step towards comprehensive analyses and policy action. Since the pioneering work by Kuznets (1955) and Atkinson and Harrison (1978), however, the topic of *systematically* collecting distributional information on wealth (and income) as an addition to macro aggregates has been evoked only recently: The *Commission on the Measurement of Economic Performance and Social Progress* emphasises in the "Stiglitz-Sen-Fitoussi report" the need to give more prominence to the distribution of income, consumption and wealth.[2] This need is also expressed in the *G20 Data Gaps Initiative*.[3] In 2011 the *World Top Incomes Database*[4] was made public and started publishing series on income inequality for several countries. In 2015

[2] Stiglitz-Sen-Fitoussi report, recommendation 4, available at `http://www.stiglitz-sen-fitoussi.fr`.
[3] Recommendation 16. Report available at: `http://www.imf.org/external/np/g20/pdf/102909.pdf`.
[4] See `http://www.WID.world` and Alvaredo et al. (2016).

the database was renamed *World Wealth and Income Database* to emphasise the particular importance of wealth, and aims to provide comprehensive and long data series on distributional indicators on income and wealth for a large number of countries around the globe.

Collecting distributional data may ultimately lead to *Distributional National Accounts* (DINA). The aim of DINA is to have distributional estimates available that are consistent with macroeconomic statistics such as national accounts (Fesseau and Mattonetti, 2013; Alvaredo et al., 2016; Piketty et al., 2016). Piketty et al. (2016) name the gap between national accounts and general studies on inequality, which usually do not aim for consistency with national accounts aggregates, as the most important limitation that needs to be overcome to rigorously measure income inequality and thus call for DINA. Due to better data availability there is already substantial progress with regard to income, whereas wealth-related indicators stand at the very beginning.

While it is well known that wealth is highly concentrated at the top, recent data show that top income and wealth shares – and in particular wealth-income ratios – have even risen in many developed and developing countries in recent decades (Alvaredo et al., 2017). However, extensive administrative data that would allow a sound analysis of these developments are hardly available in many European countries including Austria and Germany. On the one side, this is due to limited spread of wealth-related taxes[5] which disincentivises data collection. Above that, data protection laws and privacy concerns further limit the use of existing administrative data.

As administrative data is mostly not available, wealth surveys are therefore a distinct source of information leading towards the compilation of DINA. Unfortunately, macro and micro data are not, and cannot always be, completely aligned and definitions as well as concepts differ in some instances. This gave rise to research comparing micro data gained from surveys with macro data such as national accounts to analyse commonalities and differences between both sources (Kavonius and Törmälehto, 2010; Kavonius and Honkkila, 2013; Henriques and Hsu, 2014; Andreasch and Lindner, 2016; Baranyai-Csirmaz et al., 2017; EG-LMM, 2017).

While surveys have the advantage of a long list of variables compared to (scarcely available) administrative data, voluntary surveys suffer from several kinds of reporting errors including a lack of observations from the wealthiest part of the population.[6] Due to the high concentration of wealth at the very top (see Davies and Shorrocks, 2000), even more observations in this part of the distribution would be necessary to ensure an acceptable degree of precision. However, surveys and particularly wealth related surveys usually fail to capture these households appropriately as they are harder to be sampled, to be contacted, and to get hold of.

There is evidence from the Spanish (Bover et al., 2014, Table 5) and US (Kennickell and Woodburn, 1999; Kennickell, 1999a) household wealth survey that response rates decline with increasing wealth leading to a high unit-non-response rate in this part of the distribution. Such systematic non-response is likely to introduce a unit-non-response bias.[7]

Osier (2016) analyses a potential unit-non-response bias for the first wave of the *Household Finance and Consumption Survey* (HFCS). He finds that "[u]nit non-response is a key concern for the HFCS, as non-response rates are important in some countries and response patterns are

[5]For instance, Wöhlbier (2014) report that "[m]ost net-wealth taxes were removed or scaled down by [EU] Member States between 1995 and 2007."

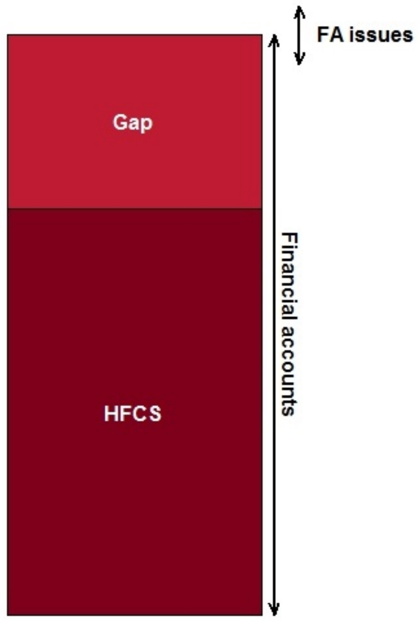
[6]Cowell and van Kerm (2015) survey measurement issues related to wealth distributions and wealth inequality estimated from household surveys.

[7]*Unit-non-response bias* refers to the bias introduced when certain households systematically do not participate or participate less often in a survey. It has to be noted, though, that in general (and in particular when the non-participation is random) low response rates must not necessarily lead to strong bias and vice versa.

not completely random" (Osier, 2016, page 3).

In general, for Austria and Germany a quantification of unit-non-response bias is problematic as no external household level data on wealth can be used for such an analysis. However, "[i]n a wealth survey, the sensitivity of the subject and the time cost of being interviewed, for people with complex assets, should be enough to raise *a priori* concerns" (Kennickell, 2008, page 405).

Figure 1: Discrepancies between FA and HFCS aggregates.



Notes: The figure illustrates the most common case for conceptually well comparable instruments: under-coverage, i.e., micro (HFCS) are lower than macro (financial accounts) aggregates.

This underrepresentation of the wealhtiest hoseuholds poses a problem when aggregates of wealth surveys are compared to macro statistics. Aggregated micro data are usually lower than macro statistics as shown in Figure 1 and one important reason are "missing wealthy households"[8] (see also Avery and Elliehausen, 1986; Avery et al., 1988) which contribute to the micro-macro gap through two channels: unit-non-response as well as a lack of precision due to too few observations at the very top. While strategically oversampling the wealthiest households in a survey aims to reduce the latter problem, in principal it is not well suited to correct for unit-non-response bias.

There have been recent attempts to analyse the impact of underrepresentation of wealthy households on the wealth distribution (Bach et al., 2014; Eckerstorfer et al., 2016; Vermeulen, 2016).[9] This is usually done by replacing observed wealth of the richest households in the

[8]Other sources of this gap include conceptual discrepancies (differences in the definition of instruments, different valuation methods), population discrepancies (different scopes of the survey and financial accounts), reporting errors (intentional under- or over-reporting and related behavioural effects, reporting errors due to a lack of knowledge; see D'Aurizio et al. 2006), rounding errors (rounding of amounts by respondents and additional rounding for the purpose of anonymisation), and sampling errors (errors in the sample design). Althoguh financial accounts are supposed to be exhaustive, they are no perfect benchmark due to, for instance, balancing across accounts, difficulties in the valuation of unquoted assets and potential coverage problems of wealth held by residents abroad (see Zucman, 2013). See EG-LMM (2017) for more details regarding population and conceptual discrepancies.

[9]Similar problems arise when estimating the income distribution from survey data. See, for instance, Törmälehto (2017) for details regarding the *European Union Statistics on Income and Living Conditions (EU-SILC)*.

survey (also referred to as the tail population) by a parametric model – such as a Pareto model – yielding a semi-parametric approach.[10]

When estimating a Pareto distribution, observations from rich lists (such as for instant the Forbes World's billionaires list) may be used to complement the survey. Rich lists add observations at the very top of the distribution, where the survey does not provide any information, and thus enhance the reliability of the Pareto model.

The goal of our paper is to quantify the contribution of the "missing wealthy" on the gap between macro and micro data for a list of specific instruments. Thereby, the term "missing wealthy" refers to both issues related to wealthy households: unit-non-response bias and a lack of precision at the top tail.

We propose two complementary approaches to achieve this goal: The *analytical approach* derives easy-to-implement formulae to compute aggregates that are adjusted for the missing wealthy. The *simulation approach* combines a Monte Carlo simulation and a stratified bootstrap procedure, and complements point estimates with empirical confidence intervals, standard errors and distributional information within the tail. As the simulation approach mirrors the analytical approach, aggregates obtained via the simulation approach converge to analytically derived aggregates.

Both methodologies consist of two parts: First, we substitute the top tail of the empirical HFCS wealth distribution by a Pareto model estimated from survey and rich list data. Using survey weights, we can derive the total number of households in the tail. This information is then used to calculate total wealth for several net worth strata in case of the analytical method and simulate Pareto-distributed net worth for each tail household in case of the simulation method.

In a second step, we break down net worth by instruments. This step relies on stratification. We use portfolios observed in the HFCS data and stratify them into four sets representing four net worth ranges. The portfolio strata are matched with corresponding ranges in the net worth distribution. While the analytical approach relies on average portfolio shares per stratum, the simulation approach matches each simulated net worth with a portfolio (i.e., shares of assets and liabilities of net worth) belonging to the same stratum.

We find that stratifying by net worth is of utmost importance as we observe strong correlation between the latter and portfolio structure – even within the wealthiest of the wealthy households.

For the simulation, we repeat the two steps of our methodology numerous times to gain reliable estimates: More specifically, we draw 100 samples from the Pareto distribution and perform the stratified bootstrap for each of the samples 100 times yielding 10,000 repetitions in total. The size of each of the samples equals the number of households in the tail population (here defined as households with net worth of at least one million Euro), and hence the number of simulated observations, on which our analysis is based on, ranges between 12.4 billion (Germany) and 1.3 billion (Austria).

We perform a case study for Austria and Germany using the second wave of the HFCS released in December 2016. We chose these countries as they lack additional administrative micro data on wealth to perform strategic oversampling of the wealthy, and correct and cross-check

[10] Another way of addressing this issue is to create artificial observations representing the wealthiest households and add them to the survey with corresponding weights. Such an approach is followed by the Hungarian Central Bank (see Baranyai-Csirmaz et al., 2017, page 53ff). These artificial observations are constructed considering information on the very rich obtained from rich lists and other external data sources such as company registers or tax data.

self-reported wealth components.[11] We thus expect larger effects for these countries. Above that, "[b]oth the composition and the distribution of wealth in Germany and in Austria exhibit considerable similarities" (Fessler et al., 2016, page 29) allowing us to compare results.

Our findings show that when relying on the HFCS only, one concludes that the richest 1% roughly hold 25% of total wealth in Austria and 24% in Germany. Replacing the top tail by a Pareto distribution estimated from the survey and complemented with data from rich lists increases this share to roughly 43% in Austria and 36% in Germany.[12]

We find that equity[13] has an increasing importance in the portfolios of the very rich and our adjustments thus have the largest impact on this instrument. In Austria, aggregates for equity increase by roughly 106% and in Germany by roughly 38%. While the survey usually leads to an under-coverage for most instruments, aggregates for equity calculated from the HFCS tend to exceed the financial accounts counterparts (see EG-LMM, 2017). HFCS aggregates increase even more after applying our methodology. This is another hint that some of the underlying instruments (in particular the value of self-employed businesses on the HFCS side, and unlisted shares and other equity on the financial accounts side) are not well comparable and differences in valuation concepts are an issue.

Our analysis further shows that changes in other instruments, namely in deposits, bonds and liabilities (loans),[14] are much less pronounced as these instruments seem to be less important for the very rich. For these instruments coverage ratios[15] increase by 0 to 23 percentage points in Austria and by 2 to 9 percentage points in Germany still leaving a significant gap which sources need further exploration. This shows that the missing wealthy explain only a rather small fraction of the macro-micro gap for financial instruments other than equity.

We consider our analysis as the first step towards the compilation of wealth-related distributional national accounts. Distributional indicators add important information to aggregates currently found in national accounts and provide deep insights into the structure and distribution of wealth within a society. We therefore also demonstrate how our methodology can be used to improve distributional national accounts figures.

While it is not new that Austria and Germany have similar degrees of net worth inequality (Fessler et al., 2016, page 29f), we find also strikingly strong similarities in (the high degree of) inequality on a more disaggregated level. For instance, a household belonging to the poorest 20% in Austria (Germany) has on average -11,342 EUR (-25,127 EUR) compared to 323,397 EUR (186,789 EUR) for a household belonging to the wealthiest 20%.

Our paper adjusts HFCS results not only in the dimension of net worth but for a range of different asset classes and liabilities,[16] and is first to quantify the impact of under-represented

[11] While Austria refrains from oversampling at all, Germany performs oversampling based on geographic but not individual wealth or income data.

[12] Shares for the top 1% after Pareto adjustments are already documented in the literature: Vermeulen (2017) uses the first wave of the HFCS and finds shares of 32%-34% for Germany and 31%-32% for Austria. Bach et al. (2015) find 33% for Germany and Eckerstorfer et al. (2016) 38% for Austria.

[13] By equity we refer on the HFCS side to the sum of DA1140 Value of self-employment businesses, DA2104 Value of non-self-employment businesses and DA2105 Shares, publicly traded, and on the financial accounts side to F.51 Equity (which is the sum of F.511 Listed Shares, F.512 Unlisted Shares and F.519 Other Equity).

[14] These instruments are of high conceptual comparability according to findings by the *ECB Expert Group on Linking Macro and Micro Data for the Household Sector* (EG-LMM, 2017).

[15] We define the *coverage ratio* (CR) for each instrument as the ratio of the HFCS aggregate over the financial accounts aggregate. In case of $CR < 1$ we speak of under-coverage and in case of $CR > 1$ of over-coverage.

[16] Chakraborty et al. (2016) are to our knowledge the only other authors relying on a semi-parametric Pareto model for a similar instrument-specific analysis. In contrast to our stratification approach, this article only computes average portfolio shares.

wealthy households in a household survey on the macro-micro gap for several instruments.

We also add to the literature by providing more detailed information on the wealth distribution and portfolio allocation of the wealthiest households. Such information is so far hardly available for Austria and Germany due to a lack of data sources.

Above that, to our knowledge, our article is first to make use of such a quantification in the compilation of DINA. We calculate instrument-specific distributional indicators, which are useful by themselves, but also aggregate them to an overall indicator for financial wealth. Instrument-specific and aggregate indicators are fully consistent with financial accounts totals.

The remainder of this article is organised as follows: First, section 3 elaborates on the distribution of net worth and describes how a Pareto model may be used to adjust the distribution at the top. The analytical approach is described in section 4 and the simulation approach in section 5. Data is presented in section 6, section 7 reports empirical results[17] and section 8 demonstrates how our findings can be used to improve the compilation of distributional national accounts. Finally, section 9 concludes. The appendix adds important technical details, performs comprehensive sensitivity analyses, and provides further numerical results.

3 The top of the wealth distribution

In an ideal world, a survey would represent the entire population, i.e., also the most affluent households. Unfortunately, surveys – and particularly wealth related surveys – usually fail to capture these households as they are less willing to participate in a wealth survey, and are harder to be contacted.

Therefore, most countries participating in the HFCS perform some kind of oversampling of wealthy households, i.e., strategically contacting more wealthy households. Oversampling leads to more observations in a particular part of the distribution than implied by the original sample design which makes it necessary to down-weight each observation in this part to guarantee correct population totals.

In general, a survey with a sophisticated sample design yields an unbiased estimate of the aggregate – also without any kind of strategically oversampling wealthy households. Oversampling the wealthy, however, may help to decrease the *variance* of the aggregate and hence increases precision. In the case of a highly skewed distribution or equivalently speaking a high degree of inequality, the upper tail largely contributes to the aggregate and thus oversampling this important part of the distribution may be very effective to get more reliable results for a single survey wave.[18]

It is important to note though, that oversampling does not remove any kind of bias resulting from unit-non-response.

Countries with comprehensive data on wealth or income (e.g., register data maintained for the sake of collecting wealth related taxes or income tax files which are used to project wealth based on investment returns, see Kennickell, 1999b) can use this extra information to strategically contact larger numbers of wealthy households. However, in Austria and Germany no such data are used due to data protection concerns. Oversampling is hence largely restricted and can only be performed based on geographic information (e.g., to sample more from wealthy municipalities or regions within cities). Austria refrains from oversampling whereas Germany relies on a regional oversampling strategy (see HFCN, 2016a; Schmidt and Eisele, 2013).

[17] All calculations are performed using the open source programming language R (R Core Team, 2017).

[18] At the same time, oversampling the wealthy without increasing the total number of observations negatively affects other parts of the distribution due to larger weights per observation outside the right tail.

Ex-post adjustments relying on parametric models are a suitable approach to address *both* issues related to the missing wealthy: precision and bias. There is evidence that the top of the wealth distribution can be well approximated by a Pareto distribution aka the "power law" (see Pareto, 1895).[19] Thus, relying on survey observations only for the less-wealthy and a Pareto model for the wealthy, i.e., a semi-parametric approach, seems to be well suited to get hold of this problem. This idea is not new and has been widely used before.[20]

In general, a Pareto model may be estimated from survey data only. As the very top of the tail is not appropriatly represented by a survey, the estimated distribution is likely to generate a too flat tail. Hence, several articles add observations from rich lists to the estimation procedure to increase confidence in the estimated tail (see Bach et al., 2014; Eckerstorfer et al., 2016; Vermeulen, 2016, 2017).[21]

3.1 The Pareto distribution

The Pareto distribution is a two-parameter distribution with cumulative distribution function (CDF)

$$F_Y(y) = 1 - \left(\frac{y}{y_0}\right)^{-\vartheta}, \quad y \geq y_0, \tag{1}$$

and density function

$$f_Y(y) = \frac{\vartheta y_0^{\vartheta}}{y^{\vartheta+1}}, \quad y \geq y_0,$$

where $y_0 > 0$ denotes the scale (or threshold) parameter and $\vartheta > 0$ the shape parameter: Decreasing ϑ yields to a reduction of probability mass at the threshold y_0 and at the same time a prolongation of the tail.

There is no universally agreed estimation procedure to determine the threshold y_0. In general, the threshold should be large enough to guarantee that observations follow a Pareto law. The threshold, however, must also be small enough so that there are enough observations above the threshold that can be used to estimate the Pareto shape parameter. For reasons of comparability, we use the same threshold for Austria and Germany, namely one million Euro. Appendix C shows that this is a suitable choice for both countries and performs robustness checks with this regard.

On the contrary, a large number of estimators for the shape parameter is suggested in the literature. When aiming for an estimator based on survey data, a method that accounts for survey weights is needed. Vermeulen (2017) suggests a pseudo maximum likelihood and a weighted regression estimator. Alfons et al. (2013) extend robust estimators for survey data.[22] In this article, we rely on two robust estimators, and Vermeulen's regression method. Additionally, we derive a robust version of the regression method based on quantile regression that is expected to be even less sensitive toward extreme observations. Details are provided in Appendix A.

[19] Atkinson (1975), Davies and Shorrocks (2000), Klass et al. (2006), Piketty et al. (2006), Cowell (2011a) and Cowell (2011b) use the Pareto distribution to model top wealth distributions.

[20] See for instance Cowell (2011a); Bach et al. (2015); Eckerstorfer et al. (2016); Vermeulen (2016, 2017).

[21] Klass et al. (2006) analyse the wealth distribution implied by the 400 richest people in the United States according to Forbes lists for various years and find that the Pareto distribution is a very well suited model for this top tail.

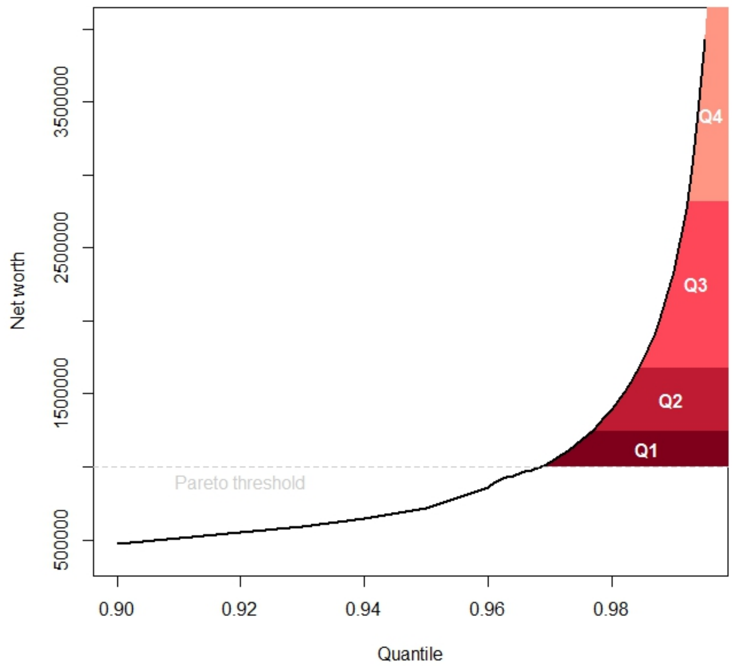
[22] They implement these estimators (namely the weighted integrated squared error (wISE) and weighted partial density component (wPDC) estimator) in the R package laeken (see Alfons and Templ, 2013).

3.2 Portfolio structure at the very top

A correlation between portfolio structures and the relative position in the wealth distribution is well documented in the literature: Wealthier households are more likely to own real estate, risky assets (such as stocks and bonds) as well as private businesses. The probability to own a private business is significantly higher for households belonging to the top 20% of the net wealth distribution (Arrondel et al., 2014).

The results of the second wave of the HFCS confirm that: While portfolios of the lower deciles of the wealth distribution consist mainly of deposits, the share of rather risky financial assets (mutual funds, bonds, publicly traded shares) increase as financial portfolios get larger (HFCN, 2016b). Appendix B provides detailed distributional information on portfolio structures calculated from second wave HFCS data for several euro area countries.

Figure 2: Illustration of the top tail of the net worth distribution.

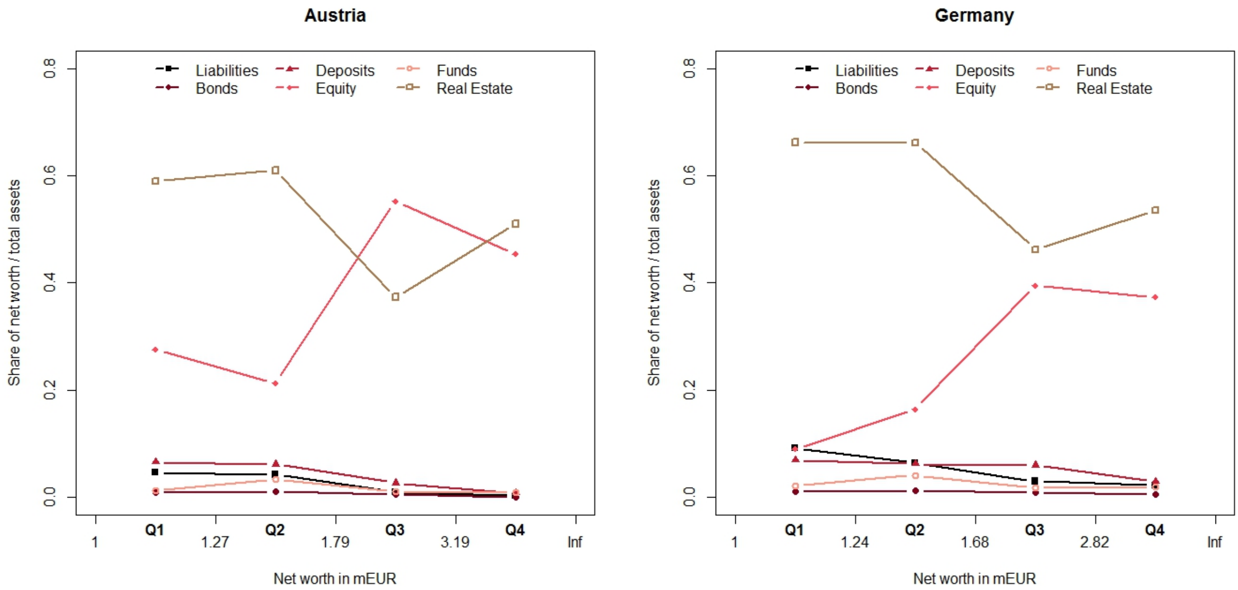


Notes: The figure depicts the top of the net worth distribution (for this illustration German HFCS data has been used). Below the Pareto threshold, the empirical distribution is plotted. Above the threshold, the parametric model, i.e., the theoretical Pareto distribution, takes over. $Q1$ to $Q4$ represent the quartiles of the tail distribution. *Source:* HFCS

We even find large variation in portfolio structures within the tail: Portfolios of the "wealthy" still differ from portfolios of the "extremely wealthy." To show that, we split up the top tail into four quartiles $Q1$, $Q2$, $Q3$, and $Q4$ as demonstrated in Figure 2.

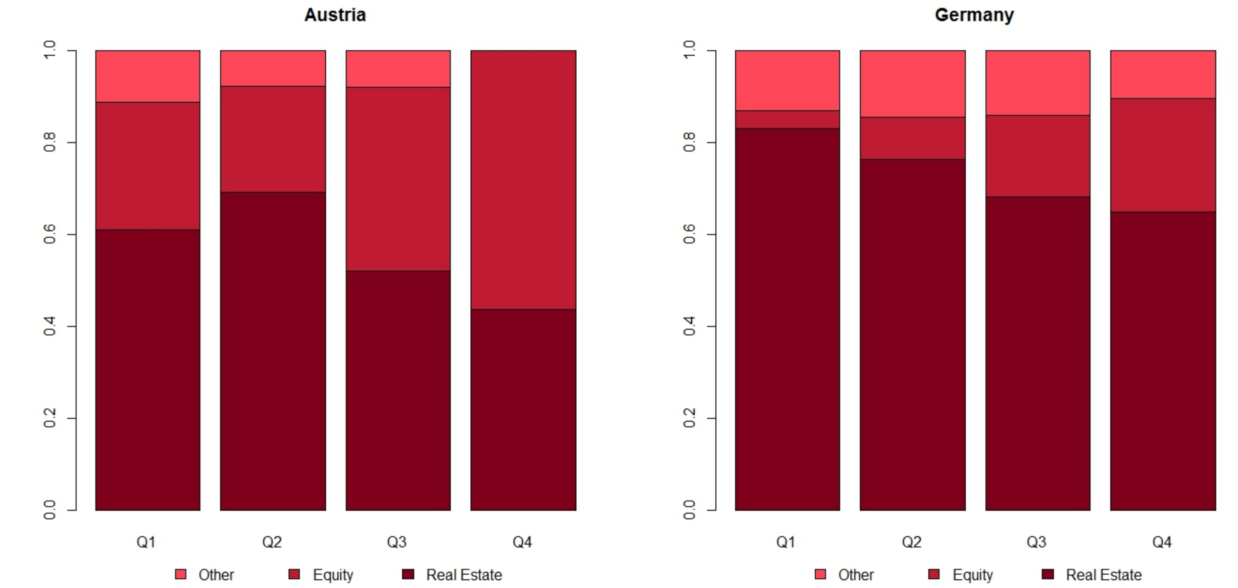
Figure 3 shows the average portfolio structure for each tail quartile: For the top 25% of the tail population, i.e., $Q4$, equity is much more important than for the rest of the tail population. On the contrary, real estate assets are more important for the lower 50% ($Q1$ and $Q2$) of the tail population. The share of total assets held as deposits, bonds, and mutual funds decreases when moving to the very top of the distribution. Figure 3 also shows the ratio of liabilities over net worth. This ratio also decreases with net worth.

Figure 3: Change in portfolio structure in the tail.



Notes: The figure shows bonds, deposits, equity, mutual funds, and real estate assets as share of total assets and the ratio of liabilities (loans) over net worth (see Table 5 for definitions). Shares are calculated separately for each net worth tail quartile (quartile thresholds are calculated from the semi-parametric Pareto model), e.g., Q4 refers to the wealthiest 25% in the tail. For the figures instrument-specific aggregates are calculated for each quartile and divided by total assets (total net worth) in the respective quartile. *Source:* HFCS

Figure 4: Types of households in the tail.



Notes: The figure shows the share of types of households in the tail split up by quartile. A household is of type "equity" when its largest position in the portfolio is equity and so forth. For Austria, results are less smooth due to fewer tail observations. *Source:* HFCS

The portfolio allocation for individual observations and for the entire tail can be found in Figure 17 in the appendix. There, each bar represents one single observation in the HFCS. The wider the bar the more households are represented by this observation. It is clearly seen that portfolio structures systematically change when moving from $Q1$ to $Q4$.

Figure 4 shows the share of "types of households" for $Q1$ to $Q4$. A household is classified to be of type "equity" when its largest portfolio position is equity and so forth. The share of "real estate households" decreases when moving up the distribution while the share of "equity households" increases. Portfolios consisting mainly of assets other than real estate and equity are generally larger in $Q1$ to $Q3$ than in $Q4$.

Although we observe clear tendencies in changes in portfolio structures which we exploit in our analysis, countries that do not oversample most probably still lack representative portfolios at the very top and accuracy of our results may be limited. Kennickell (2008) finds that for instance in the 2004 *US Survey of Consumer Finances* out of roughly 400 interviewed households that had direct holdings of government or commercial bonds, approximately 90% of these cases entered the survey through oversampling. Thus, we demonstrate in Appendix C how additional portfolios representing the wealthiest households can be used to better reflect them in the analysis but also show that there are only marginal changes to our general results.

4 An analytical approach to adjust for the missing wealthy

In the case of $\vartheta > 1$ (which holds true for all our estimations) the Pareto distribution has a finite first moment, i.e., the mean exists and is given by

$$E(Y) = \frac{\vartheta}{\vartheta - 1} \cdot y_0. \tag{2}$$

In that case, the total tail wealth can be estimated by multiplying the number of households belonging to the tail N_{tail} with the mean, i.e.,

$$N_{tail} \cdot \frac{\vartheta}{\vartheta - 1} \cdot y_0.$$

In a survey setting, N_{tail} equals the sum of weights of all households with a net worth greater than y_0.

The analytical method is straight-forward and easy to implement. However, it comes at the cost of loosing micro-structure: the dependency among instruments (as each instrument-specific aggregate is calculated independently of all other instruments) as well as the valuable variability resulting from using individual rather than average portfolio shares are lost. Calculating, for instance, the share of total assets (or liabilities) held by the top 1% or distributional measures such as Gini coefficients is not possible.

As input, the analytical method needs a Pareto model for the top tail. This model is ideally derived by combining survey and rich list observations as described in section 3. We split the Pareto tail into four strata[23] as shown in Figure 2.

We calculate average wealth for each stratum Q. Therefore, we calculate *expected net worth* Y *conditional on net worth realising in stratum* Q. Strata are defined via quartiles, which are denoted by $F^{-1}(p)$. Thus, strata are intervals $Q = [F^{-1}(p_1), F^{-1}(p_2))$, with $\{p_1, p_2\} \in$

[23] Four strata defined via quartiles seem appropriate for our case study. However, the method can easily be adjusted if a finer split-up is needed.

$\{\{0, 0.25\}, \{0.25, 0.5\}, \{0.5, 0.75\}, \{0.75, 1\}\}$. Stratum-specific expected wealth is given by[24]

$$W(Q) = E(Y|Y \in Q) = \frac{\vartheta y_0}{(1-\vartheta)\cdot(p_2 - p_1)}\left[(1-p_2)^{1-1/\vartheta} - (1-p_1)^{1-1/\vartheta}\right].$$

Total stratum-specific net worth is thus given by

$$TW(Q) = (p_2 - p_1)\cdot N_{tail}\cdot W(Q).$$

Note that in case of a split-up by quartiles $p_2 - p_1 = 0.25$.

As a next step, we calculate stratum-specific portfolio shares. Let w_k denote the weight of household k and s_j^k assets held by household k in the form of instrument j over net worth. *Instrument- and stratum-specific tail aggregates* are then given by

$$A_j(Q) = \left(\sum_{k\in Q} s_j^k \cdot \frac{w_k}{\sum_{l\in Q} w_l}\right)\cdot TW(Q).$$

Total instrument-specific aggregates are obtained by summing up stratum-specific tail aggregates and adding them to the non-tail aggregate $A_j(NT)$, which is calculated respecting survey weights:

$$A_j = A_j(NT) + \sum_{Q\in\{Q1,Q2,Q3,Q4\}} A_j(Q).$$

5 A simulation approach to adjust for the missing wealthy

The fact that the top tail of an empirical distribution is replaced by a parametric model can also be exploited for simulation. A simulation creates micro-data files that are adjusted for the too flat tail. These micro-files allow one to analyse distributional patterns at a disaggregated level. The simulation, which in fact combines Monte Carlo simulation and bootstrapping, perfectly mirrors the analytical approach, and thus empirical confidence intervals supplement the analytical results. The proposed algorithm aims to recover observed portfolio structures and thus also enables analyses with this regard.

We combine the semi-parametric model for net worth with a wealth-stratified bootstrap algorithm to conserve the correlation between portfolio structure and net worth. The bootstrap algorithm is a fully non-parametric[25] procedure.

For each household in the tail total net worth is simulated from the Pareto model. The household is then allocated to its respective net worth stratum and a suitable portfolio is drawn.

Again, we stratify tail observations into four strata[26] ($Q1$ to $Q4$) depending on their position in the net worth distribution. So, a household with simulated net worth of roughly one million Euro is allocated to the lowest quartile and thus a portfolio is drawn from the $Q1$ stratum. In contrast, households with large simulated wealth are assigned a portfolio structure from $Q4$.

[24]The derivation of this formula is given in Appendix D.

[25]The correlation between portfolio structures and net worth does not seem to be linear but rather follows distinct functional patterns. Hence, relying on a parametric model such as a Beta regression (The Beta distribution is ideally suited to model shares as it is bounded between 0 and 1.) might (and in fact does) heavily depend on the exact model specification. Semi-parametric approaches with data driven functional forms (such as Generalized Additive Models) would be an obvious candidate to overcome this issue. However, the observed tail is very sparsely populated and it was (at least in our analysis) impossible to estimate stable and trustworthy functional forms.

[26]The algorithm works for any stratification as long as there are sufficient observations per stratum. The choice here is in accordance with the analytical method.

Table 1: Simulation approach.

Step 1.	For a given y_0 estimate the shape parameter of the Pareto distribution ϑ using the combined sample of survey and rich list observations.
Step 2.	Calculate quartiles of the tail implied by the estimated Pareto distribution yielding strata $Q1$, $Q2$, $Q3$, and $Q4$.
Step 3.	Allocate each observation in the survey with a net worth of at least y_0 to its respective stratum $Q1$, $Q2$, $Q3$, or $Q4$.
Step 4.	Draw N_{tail} random numbers from Pareto$(y_0, \hat{\vartheta})$ and allocate them to the strata $Q1$, $Q2$, $Q3$, or $Q4$.
Step 5.	For each simulated net worth draw a random portfolio allocation from observed allocations in the respective stratum $Q1$, $Q2$, $Q3$, or $Q4$ respecting survey weights.
Step 6.	Calculate instrument-specific aggregates for the tail population.
Step 7.	Add the simulated tail aggregates to the non-tail aggregates and calculate instrument-specific coverage ratios.
	B observations.
Step 8.	Repeat steps 5 to 7 B times.
	$M \times B$ observations.
Step 9.	Repeat steps 4 to 8 M times.
	$N_{tail} \times M \times B$ observations.

The algorithm consists of nine steps and is summarised in Table 1. The stratified bootstrap is repeated B times for each of the M draws from the Pareto distribution each of size N_{tail}. Our choices of $M = 100$ and $B = 100$ yield in total $M \times B = 10,000$ repetitions. In the case of Germany, the analysis is thus based on $N_{tail} \times M \times B = 12,370.37$ million and in the case of Austria $N_{tail} \times M \times B = 1,264.58$ million simulated portfolios.

Appendix A describes how to calculate quartiles from a Pareto distribution (step 2) and how to sample from a Pareto distribution (step 4).

In step 5, we *draw a random portfolio allocation*. In this context, this means that we draw a full list of shares of assets/net worth but no amounts. Specifically, this list includes the ratio of liabilities over net worth, as well as the share of total assets invested in bonds, deposits, equity, and real estate. The portfolio is then constructed by, first, calculating the amount of liabilities by multiplying its drawn ratio with simulated net worth and, second, using the Euro amount of liabilities to calculate total assets. Third, we multiply total assets with the drawn share of bonds, deposits, equity, and real estate to receive the respective Euro amounts.

Thus, although we replicate observed patterns in terms of portfolio structure, simulated amounts are (on average) larger as net worth is drawn from a Pareto model which generally increases total tail wealth.

6 Data

The analysis makes use of three different data sources: First, second wave HFCS data for Austria and Germany. Second, we use aggregates from financial accounts (FA) as benchmark measures. Third, we use national rich lists that yield information on net worth of the wealthiest individuals/families.

Table 2 reports the codes used in the HFCS and financial accounts as defined in the *European System of National and Regional Accounts* (ESA2010).

Table 2: HFCS and FA codes.

HFCS codes	
DA1140	Value of self-employment businesses
DA1400	Real estate (incl. property used for business activities)
DA2101	Deposits (Value of sight and savings account)
DA2102	Mutual funds, total
DA2103	Bonds
DA2104	Value of non-self-employment businesses
DA2105	Shares, publicly traded
DL1000	Total outstanding balance of household's liabilities
DN3001	Net wealth, excl. public and occupational pensions
FA codes	
F22	Transferable deposits
F29	Other deposits
F3	Debt securities
F4	Loans (long-term and short-term)
F51	Equity
F52	Investment fund shares/units

Source: HFCS, ESA2010

6.1 HFCS data

The HFCS collects household-level data on households' finances and consumption. The second wave, which was released in December 2016, was conducted in 18 euro area countries as well as the non-euro area countries Hungary and Poland in a harmonised way while still leaving room for a country-specific implementation.[27] While some European countries have a long tradition in conducting wealth related household surveys, a harmonised survey for such a large number of countries is a recent development and results of the first wave were only released in 2013. In Austria and Germany[28] the HFCS is the first comprehensive household survey on wealth, which is why there is yet very limited knowledge about the wealth distribution in these countries.

The variables in the HFCS follow common standards and definitions. Therefore, the variables used in the analysis (assets and liabilities)[29] are in principle comparable between Austria and Germany. In both countries the survey has been mainly conducted via *Computer Assisted Personal Interviews*.

The HFCS uses probability sampling and the sample size is representative both at the country and at the euro area level. For Austria, the gross sample size was 6,308 yielding 2,997 observations (response rate of 49.8%) representing 3.9 million households while in Germany the gross

[27]See HFCN (2016a) for a general documentation and `https://www.hfcs.at/en/` (Austria) and `http://www.bundesbank.de/Navigation/EN/Bundesbank/Research/Panel_on_household_finances/panel_on_household_finances.html` (Germany) for a more detailed country-specific documentation.

[28]In Germany, the HFCS is labelled *Panel on Household Finances* (PHF).

[29]We frequently use the financial accounts terminology and use the word "instruments" rather than "items" or "variables" as often used in a survey context.

Table 3: HFCS data.

AT – Austria					
	Total	Summary Statistics for the tail			
	Aggregate	25%	Mean	Median	75%
Net worth	998,130	1.220	2.618	1.518	2.040
+Liabilities	66,601	0.000	0.044	0.000	0.042
Total Assets	1,064,730	1.284	2.662	1.609	2.068
Bonds	5,273	0.000	0.012	0.000	0.000
Deposits	98,745	0.028	0.089	0.055	0.100
Equity	194,739	0.000	0.942	0.217	1.000
Mutual Funds	17,070	0.000	0.044	0.000	0.006
Real Estate	686,174	0.458	1.479	0.910	1.300
Number of observations:					2,997
Number of observations in the tail:					85
Number of households:					3,862,526
Number of households above 1 mEUR:					126,458
Total population:					8,543,930
Average number of persons per household:					2.212

DE – Germany					
	Total	Summary Statistics for the tail			
	Aggregate	25%	Mean	Median	75%
Net worth	8,500,090	1.202	2.611	1.570	2.472
+Liabilities	1,020,920	0.000	0.118	0.000	0.127
Total Assets	9,521,010	1.267	2.729	1.654	2.574
Bonds	71,916	0.000	0.055	0.000	0.005
Deposits	1,007,795	0.021	0.187	0.070	0.200
Equity	1,326,078	0.001	0.735	0.087	0.500
Mutual Funds	206,863	0.000	0.082	0.000	0.037
Real Estate	5,872,317	0.715	1.487	1.065	1.650
Number of observations:					4,461
Number of observations in the tail:					379
Number of households:					39,672,000
Number of households above 1 mEUR:					1,237,037
Total population:					80,983,000
Average number of persons per household:					2.041

Source: HFCS, Eurostat (population totals for 2014)

Notes: Amounts are in mEUR. 25% and 75% refer to the first and third quartile, i.e., the 25% and 75% percentile.

sample size was 16,221 yielding 4,461 observations (response rate of 19%)[30] representing 39.7 million households.

Table 3 provides HFCS summary statistics. Specifically, details are provided for the wealthiest households here defined as all households with a net worth of at least one million Euro.

Most countries that conduct the HFCS try to oversample the wealthy as it is known that their influence on total wealth is substantial and these households are less likely to participate in

[30]Response rate considering households that are interviewed for the first time. (Germany has a panel component.) The response rate including panel households is 29% (see HFCN, 2016a).

surveys. The success of an oversampling strategy can be measured by the *effective oversampling rate* (see also HFCN, 2016a), which is defined for the top $x\%$ as

$$\frac{S(1-x)-x}{x},$$

where $S(1-x)$ is the share of sample households in the wealthiest $x\%$.

For our analysis, the oversampling strategy is of particular relevance as we focus on the top part of the wealth distribution. Austria refrains from oversampling wealthy households which results in an effective oversampling rate of -33% for the top 1% while Germany uses regional indicators for their oversampling which results in an effective oversampling rate of 131% for the top 1%.[31] As we mainly analyse households with a net worth of at least a million Euro we also report the effective oversampling rate of millionaires: For Germany, this rate equals 171% while for Austria -14%.[32]

The HFCS relies on multiple imputation to fill in gaps due to item-non-response, and erroneous and implausible entries.[33] Imputations are not free of doubt which is why five implicates are provided. For our analysis, for each imputed value we use the average over all five implicates.

Our methodology critically depends on an exact estimate of the number of households owning at least one million Euro. We estimate this number from the HFCS by summing up the weights of the wealthiest households. Results are also provided in Table 3. According to the *World Wealth Report* (Capgemini, 2016) there are 114,000 (121,000) *high net worth individuals* (HNWIs)[34] in 2014 (2015) in Austria and 1,141,000 (1,199,000) in Germany. Although the definitions of HNWIs and millionaires in the HFCS are not perfectly aligned, the numbers can still be used as a benchmark. The very similar results increase our confidence in HFCS numbers.

6.2 Financial accounts data

Aggregates compiled under the ESA2010 framework provide a detailed, consistent and generally exhaustive description of an economy. Within the European Union data is well comparable as Member States are legally obliged to follow common accounting rules.

For our comparisons, we use financial accounts data which form an integral part of the ESA2010 accounting framework.

Aggregates from financial accounts are reported in Table 4. The period of measurement in financial accounts are either quarters or years. As fieldwork periods of the HFCS are usually longer than a quarter but shorter than a year, we calculate weighted averages of quarterly financial account data using all quarters that overlap with the respective fieldwork period: If, say, the fieldwork period was three months of which one month falls into quarter A and two months fall into quarter B, then quarter A will be weighted with 1/3 and quarter B with 2/3.

[31] If e.g. the share of very wealthy (top 1%) households in the net sample is exactly 1%, then the effective oversampling rate of the top 1% would be zero. In general, oversampling is thus successful when the effective oversampling rate is greater than zero. In the case of Austria, due to unit-non-response this rate is even negative (i.e., there are even less wealthy households in the net sample then there would be if all households had the same weight) and thus leads to an up-weighting of observations at the top increasing the influence of every single one. This has potentially a negative effect on the precision of estimates.

[32] We calculate this rate by empirically inverting the empirical cumulative distribution function implied by HFCS data. For Austria this yields in fact the 0.0330 rate and for Germany the 0.0313 rate.

[33] When a respondent generally agrees to participate in the survey but denies to answer some particular questions, these missing values are classified as item-non-responses.

[34] HNWIs are defined as individuals having at least one million USD of investable assets, excluding primary residence, collectibles, consumables, and consumer durables.

Table 4: Financial accounts data.

AT – Austria					
HFCS fieldwork period:					June 2014 – February 2015
Number of months:					9
	Q2:2014	Q3:2014	Q4:2014	Q1:2015	Weighted
Quarter weight	0.111	0.333	0.333	0.222	Average
Liabilities	165,866	167,271	167,936	171,543	168,286
Bonds	43,058	41,869	40,476	38,860	40,868
Deposits	213,652	212,285	217,867	219,472	215,895
Equity	120,586	120,241	127,252	131,278	125,069
Mutual Funds	45,186	46,549	47,787	51,984	48,018

DE – Germany					
HFCS fieldwork period:					April 2014 – November 2014
Number of months:					8
	Q2:2014	Q3:2014	Q4:2014	Q1:2015	Weighted
Quarter weight	0.375	0.375	0.250	0	Average
Liabilities	1,556,417	1,566,132	1,570,512	1,572,707	1,563,584
Bonds	176,357	168,895	162,198	156,766	170,019
Deposits	1,822,186	1,835,383	1,870,435	1,880,884	1,839,197
Equity	502,825	497,196	508,914	563,395	502,236
Mutual Funds	420,608	431,679	442,504	487,658	430,234

Sources: Oesterreichische Nationalbank, Deutsche Bundesbank

Notes: Numbers are in mEUR. Quarters are weighted according to the fieldwork period. Data is for the household sector only, i.e., S.14 according to ESA2010.

6.3 Comparability of HFCS and FA instruments

Although the definitions of variables are similar in both the HFCS and FA, they are not fully comparable. The *ECB Expert Group on Linking Macro and Micro Data for the Household Sector* (EG-LMM) developed a catalogue that assesses the conceptual comparability for each individual instrument (see EG-LMM, 2017). Their findings are summarised in Table 5.

Table 5: Conceptual comparability of HFCS and FA instruments.

	FA instrument	HFCS instrument	EG-LMM classification
Liabilities	F4	DL1000	High
Bonds	F3	DA2103	High
Deposits	F22 + F29	DA2101	High
Equity	F51	DA2104 + DA1140 + DA2105	Medium
Mutual funds	F52	DA2102	High
Real estate	–	DA1400	–
Net worth	–	DN3001	–

Notes: The table reports conceptual comparability as assessed by the EG-LMM (2017). See Table 2 for definitions.

Whereas liabilities, bonds, deposits, and mutual funds are in principal highly comparable, the case is more complicated for equity.[35] While listed shares as part of equity are highly comparable between the HFCS and ESA2010, other parts of equity are not. In particular, there is a valuation discrepancy for unlisted shares and other equity / (non) self-employment businesses: The HFCS relies on self-evaluation of the market value.[36] In financial accounts, "the valuation for unlisted shares and in particular holdings of other equity is less accurate [than for items with quoted market prices] as their valuation requires assumptions and modelling. In the case of unlisted shares, the valuation should mirror that of similar listed shares, the value of own funds, or discounted expected profits. Other equity reflects the value of own funds (conceptually calculated as assets minus liabilities, in practice only book values may be available)" (EG-LMM, 2017, page 8).

We look only at the aggregate of equity since the EG-LMM assessed the comparability of the more detailed break down, i.e., into unlisted shares and other equity, as low.

In general, we refrain from directly comparing net worth between the HFCS and FA. Chakraborty et al. (2016) and EG-LMM (2017) analyse different wealth concepts and conclude that at this early stage such a comparison suffers from many conceptual difficulties.

Similarly, we do not link real estate assets yet. This comparison will be possible once data coverage in the national accounts is improved.[37]

Another important issue concerning the comparability between HFCS and FA data (see EG-LMM, 2017, page 17f for more details), emerges from the distinction of producer households (part of the household sector), and quasi-corporations and corporations (part of the non-financial corporations sector) in the system of national accounts. This distinction affects the composition of the household sector balance sheet. In general, if an unincorporated enterprise is considered to be a separate unit, it is recorded in the corporate sector and shares held by households are recorded as other equity on a net basis. In contrast, unincorporated enterprises classified as producer households are part of the household sector, and their assets and liabilities are spread over all instruments. In the HFCS, a concept comparable to producer households does not exist. Assets and liabilities of self-employed businesses are recorded as net values. From this aspect alone, we would thus expect over-coverage for equity and under-coverage for remaining instruments.

6.4 Rich lists

In many countries, journalists create lists of the wealthiest individuals and families, of which the *Forbes World's Billionaires List* (e.g., Forbes, 2015) is probably the most famous one. Forbes lists the name and net worth of all individuals and families owning at least one billion US-dollars every year. It uses a number of different sources and methodologies to estimate the wealth of the financial elite around the globe.

[35] Equity as defined here differs from the HFCS definition of "business wealth," which includes properties used for business purposes. These assets are added to non-business real estate here and show up as part of "real estate." In Austria and Germany, the variable DA1140 also includes land and buildings being part of small farms (excluding the part used as main residence). With this regard the distinction between business real estate and other business wealth is not perfect.

[36] HFCS question: "What is the net value of your /your household's share of the business? That is, what could you sell it for, taking into account all (remaining) assets associated with the business and deducting the (remaining) liabilities?"

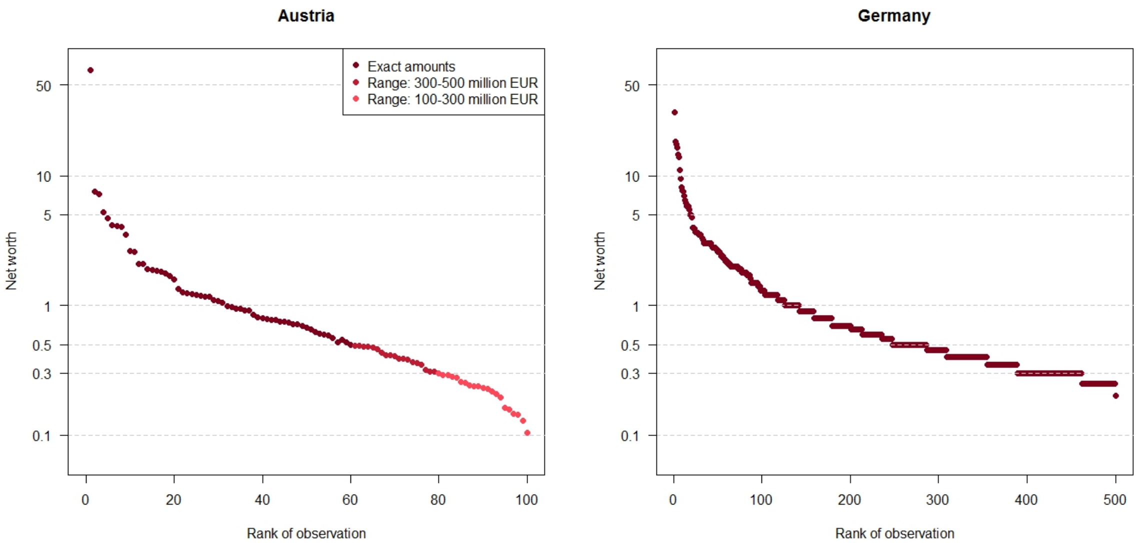
[37] The *ESA Transmission Programme* requires the transmission of annual data on land only by end-2017. The EG-LMM has not yet made a final decision on the conceptual comparability, but expects it to be at least of medium comparability. In general, national accounts separate real estate into AN.111 Dwellings, AN.112 Other buildings/structures and AN.211 Land.

Above that, there are several national rich lists. For Austria, the weekly business magazine *Trend* (trend, 2015) publishes a list of the wealthiest 100 Austrians every year. For Germany, the *Manager Magazine* (manager magazine, 2014) publishes the list of the wealthiest 500 Germans.

In this article, we rely on national rich lists. National lists are more comprehensive as they list not only billionaires. For instance in the case of Austria, there are only roughly ten observations on the Forbes list per year, whereas the Trend list consists of 100. Additionally, the Trend and Manager Magazine lists are measured in Euro, which – compared to the Forbes list measured in USD – does not introduce an additional source of ambiguity due to changes in exchange rates.[38]

Next to various measurement issues there is another major problem when using rich lists to adjust the tail of the wealth distribution: The Forbes, Trend, and Manager Magazine list fail to distinguish between households and family clans, whereas both the HFCS as well as financial accounts use households as unit of recording. This is why we do not directly rely on exact amounts reported in the rich lists but rather use these observations only to increase the quality of our Pareto estimates. We also perform some robustness checks with this regard as reported in Appendix C.

Figure 5: Rich lists.



Notes: The figures plot observations from rich lists. Net worth is in billion Euro. Due to the high degree of skewness, net worth is plotted on a log-scale. For Austria, amounts of less than 500 million Euro are reported as ranges in the Trend list and are here distributed uniformly within the range to avoid clusters. *Sources:* HFCS, Trend, Manager Magazine.

Trend list: The magazine lists the names, net worth and major sources of wealth of the richest individuals or families in Austria. Additionally, they classify the "type" of assets: assets hold in the form of a private foundation ("Stiftungsvermögen"), listed and unlisted shares ("Betriebsvermögen"), and inherited assets. This classification shows that in Austria the wealthy

[38] The number of Austrian names on the Forbes list changed from eleven in 2014 to eight in 2015 which is probably at least partly due to a drop in the exchange rate.

store large parts of their assets in the form of private foundations.[39] We perform robustness checks (see Appendix C) on how estimates change when leaving out rich list observations that hold their assets exclusively as foundations.

For individuals with net worth of less than 500 million EUR, Trend does not report the exact amount but only a range. Altogether there are 60 observations with exact amounts, 19 observations with a net worth between 300 and 500 million, and 21 observations with net worth ranging between 100 and 300 million EUR. To avoid cluster effects in the estimation of the Pareto tail, we assume that the net worth of range observations is uniformly distributed within the respective range and assign them an accordingly drawn random number (the result is shown in Figure 5).

Like Forbes, Trend uses a large number of different sources to compile their rich list. The list is published in June. We use the data from the 2015 list, as this best matches the fieldwork period in Austria.[40]

Manager Magazine list: The Manager Magazine publishes the list of the wealthiest 500 individuals and families in Germany together with their major source of wealth (e.g., real estate assets or the name of a company). Like the Forbes and Trend list, a wide range of sources and methodologies are used when compiling the list. In contrast to the Trend list, "exact" amounts[41] are reported rather than ranges and therefore no adjustment has to be made with this regard. The list does not provide information about the type of assets.

As the field-work period for Germany fully falls into the year 2014, we make use of the 2014 list.[42]

7 Empirical application

7.1 The adjusted wealth distribution

Pareto estimation results depend on the choice of method to estimate the shape parameter. Remember that in this article we rely on four different estimation techniques described in Appendix A. We choose the estimation procedure according to graphical inspection.[43]

Figure 6 shows these choices graphically and Table 6 reports the respective numbers. For both countries, the shape parameter is chosen according to the regression method as it seems to be best suited to capture the richest households appropriately.

[39] We see many private foundations ("Privatstiftungen") in Austria among the rich list as foundations serve as a way to secure the wealth of the family (e.g., the founder of a company can prevent fragmentation of business assets) and a private foundation can benefit from tax-preferred treatments (e.g., to reduce inheritance taxes as well as taxes on capital income). The majority of private foundations in Austia follow private purposes and not charitable or public purposes (Schneider et al., 2010). The claims by households in these private foundations are in the national accounts recorded in the household sector (see Andreasch et al., 2015). The national questionnaire of the Austrian HFCS has a dedicated question asking for the value of private foundations, but in the first two waves only one household has been interviewed that has a private foundation and this household did not provide any value. Therefore adding information from the rich list in Austria is particular important and complements the HFCS rather well. The analysis by Andreasch et al. (2015) on equity stakes held by private foundations furthermore indicates that wealth in private foundations is rather concentrated even inside the group of private foundations showing similar patterns as the rich list and a Pareto-like behaviour.

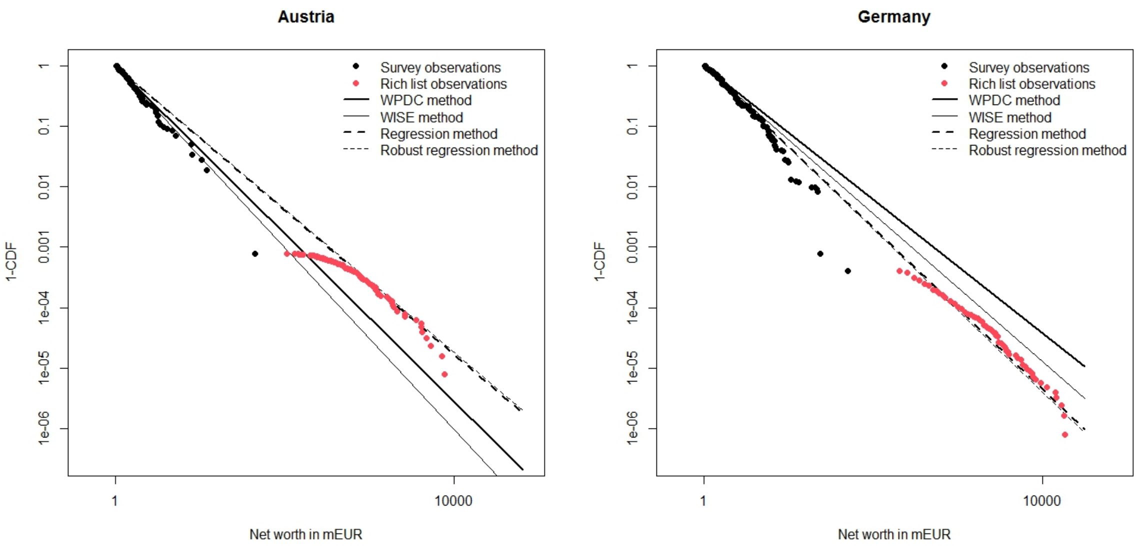
[40] For listed shares, they use values as of the beginning of May 2015.

[41] Net worth is estimated and thus cannot be "exact." Above that, amounts are heavily rounded.

[42] For listed shares, they use values as of September 2014.

[43] From theory, we know that the logged random variable to be modelled (here: net worth) and the log of one minus the cumulative distribution function (CDF) have a linear relationship. Thus, we look for a straight line that best fits our observations. This is shown in Figure 6.

Figure 6: Estimated Pareto distributions.



Notes: The figures plot the (empirical) complementary cumulative distribution function (which is defined as 1-CDF) on a log-log scale. Survey observations as well as observations from rich lists are included. The graphs are used to decide which estimation method is most appropriate for each country. *Sources:* HFCS, Trend, Manager Magazine

Table 6: The choice of shape parameter.

	AT – **Austria**		**DE** – **Germany**	
	$\hat{\vartheta}$	Implied tail wealth	$\hat{\vartheta}$	Implied tail wealth
wISE	1.505	376,951	1.225	6,743,922
wPDC	1.389	451,950	1.107	12,853,001
Regression method	1.194	778,285	1.338	4,899,939
Robust regression method	1.186	808,018	1.350	4,767,989

Notes: The implied tail wealth is calculated using formula (2) and is reported in mEUR. Estimation is performed for the combined survey and rich list data.

We draw $M = 100$ samples each of size N_{tail} from the Pareto$(y_0, \hat{\vartheta})$. Table 7 analyses the distribution of net worth when replacing the observed tail by samples from the Pareto distribution. Average results of all $M = 100$ replicates as well as coefficients of variation (CoV) are reported. Additionally, the table reports results when exclusively relying on survey data.

For both countries, the semi-parametric Pareto model leads to higher degrees of wealth inequality compared to calculations relying on survey data only. In Austria, the top 1% is estimated to own roughly 43% of total wealth whereas when relying on the survey only, this number is

Both, the wPDC and wISE estimators are desigend to be robust against outliers. As all our rich list observations are somewhat "outliers" these procedures take these observation into account to a lesser degree. The regression and robust regression method are succesful to combine the information from the HFCS as well as the rich lists and lead to almost indistinguishable results. Vermeulen (2017) compares several other methods and also concludes that the regression method including observations from the Forbes list is suited best.

[44] To plot the Lorenz curve, among the 100 simulated Pareto samples the one is chosen which fits best the theoretical Pareto distribution according to a Kolmogorov-Smirnov test. Gini coefficients reported here are the average coefficient over all 100 replicates.

Table 7: Pareto estimation results.

AT – Austria	Simulation Mean	Simulation CoV	Survey	Change
Total wealth	1,377,072	0.186	998,130	38.0%
Total tail wealth (simulation)	744,985	0.344	366,043	103.5%
Total tail wealth (analytical)	778,285		366,043	112.6%
Share of tail wealth of total wealth	54.1%		36.7%	17.4 pp
99th Percentile	2.70	0.003	1.94	39.2%
Wealth of top 1%	608,560	0.421	247,125	146.3%
Share of top 1% of total wealth	43.3%		24.8%	19.4 pp
Gini coefficient	79.6%	0.022	72.4%	7.2%
Effective oversampling rate				
of the top 5%	-11.9%		-11.9%	± 0 pp
of the top 1%	-49.9%		-13.2%	-36.7 pp

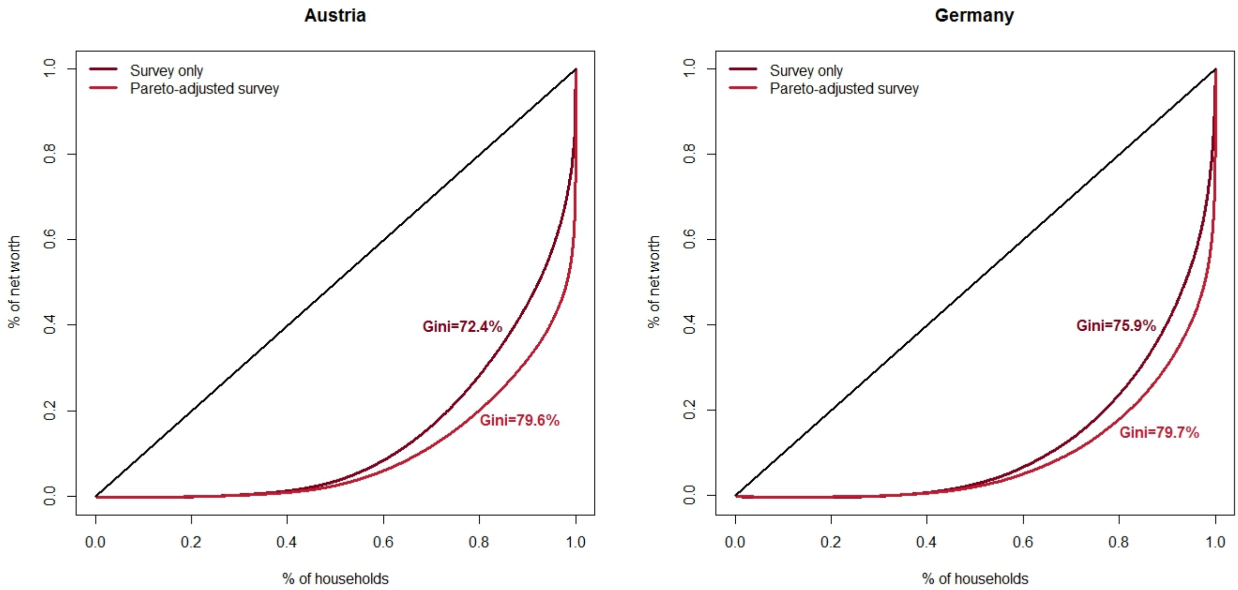
DE – Germany	Simulation Mean	Simulation CoV	Survey	Change
Total wealth	10,090,679	0.017	8,500,090	18.7%
Total tail wealth (simulation)	4,867,181	0.036	3,276,592	48.5%
Total tail wealth (analytical)	4,899,939		3,276,592	49.5%
Share of tail wealth of total wealth	48.2%		38.5%	9.7 pp
99th Percentile	2.34	0.001	2.34	0.2%
Wealth of top 1%	3,644,236	0.048	2,059,921	76.9%
Share of top 1% of total wealth	36.1%		24.2%	11.9 pp
Gini coefficient	79.7%	0.004	75.9%	3.8 pp
Effective oversampling rate				
of the top 5%	175.3%		175.3%	±0 pp
of the top 1%	130.9%		130.9%	±0 pp

Source: HFCS

Notes: Amounts are in mEUR. Shares (e.g., the top 1%) refer to the share of *households* but not individuals. Likewise, the Gini coefficient is calculated on the household level. CoV refers to the coefficient of variation, i.e., the standard deviation divided by the mean. *Tail* refers to households with a net worth of at least one million EUR. *Analytical* refers to the theoretical mean using the estimated parameters and formula (2). Simulation results are based on $M = 100$ draws from the respective Pareto distribution. "pp" refers to percentage points.

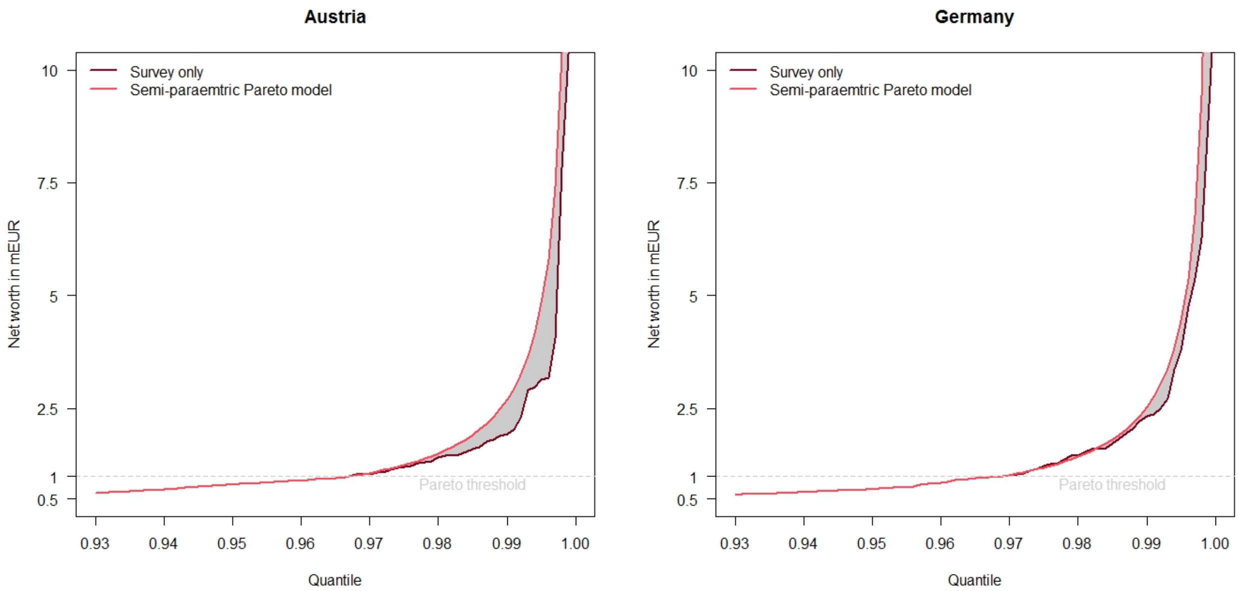
estimated to be 25%. In the case of Germany, this kind of inequality increases from 24% to 36%. Together all millionaires in Austria own roughly 54% of total wealth (compared to 37% when relying on the survey only) and in Germany 48% (compared to 39% when relying on the survey only). Likewise, household-level Gini coefficients increase from 72% to 80% in Austria and from 76% to 80% in Germany. Figure 7 shows household-level Lorenz curves before and after adjusting for the missing wealthy. The Lorenz curve is read as follows: The poorest x%

Figure 7: Lorenz curves for net worth.



Notes: The figures show household-level Lorenz curves for net worth before and after adjusting[44] for the missing wealthy. Survey weights are respected. *Source:* HFCS

Figure 8: Top tail of the wealth distribution.



Notes: The figure plots the top tail of the Austrian and German net worth distribution once estimated using the HFCS data only and once using the semi-parametric Pareto model, i.e., the combination of observed survey data and a theoretical Pareto tail estimated from survey and rich list observations. *Source:* HFCS

of the households hold y% of total wealth.

These findings are in-line with prior analyses based on HFCS data of the first wave,[45] which

[45]The fieldwork for the first wave was carried out in 2010 and 2011.

all find that adjusting the tail for the missing wealthy pushes up net worth and in particular the worth at the top. Eckerstorfer et al. (2016) find for Austria that aggregate wealth increases by 28 percentage points. The share of the richest 1% increases by 15 percentage points to 38%. Vermeulen (2016) finds that the share of the top 1% increases by 6-7 percentage points in Germany and by 8-11 percentage points in Austria when adding observations from the Forbes list. Figure 8 plots the shift of the distribution when replacing the top tail by a Pareto model. For both countries, mainly the top 1-2% are affected.

Observations from rich lists can also be used to assess the success to estimate the wealth distribution. According to the rich lists, there are 31 billionaires in Austria (according to the Forbes list, in 2014 there are 11 and in 2015 only eight USD-billionaires) and 142 in Germany. In the survey, however, not a single billionaire is interviewed indicating that the survey really fails to capture this – in terms of aggregate wealth – very important part of the population. When drawing from the Pareto distribution, we create on average eight billionaires in Austria and 121 in Germany, which matches the number on the rich lists reasonably well.

Simulated observations can also be used to assess the effective oversampling rate of the wealthy: Austria is one of the very few countries that do not oversample at all. The effective oversampling rate is corrected downwards for Austria when adjusting for the missing wealthy. Results are reported in Table 7. For Austria, the effective oversampling rate is even negative, which means that there are less observations in the top tail than implied by the sample design.

7.2 Effects on instrument level

While the effect on net worth and the overall wealth distribution can be somewhat expected when Pareto estimates are used, the effect on instrument-specific coverage ratios and instrument-specific distributions is less straightforward. In general, we find that coverage ratios increase when substituting the top tail by a Pareto model thus confirming the hypothesis of under-representation of the most affluent households in the survey.

Generally, we report point estimates calculated via the analytical method. Any additional distributional information as well as confidence intervals are obtained from the simulation method.

Table 8 summarises the main results and Figure 9 graphically shows the changes in coverage ratios for highly comparable instruments. In general, coverage is higher for Germany than for Austria. In Austria, all coverage ratios increase except for bonds where the coverage ratio remains unchanged.[46]

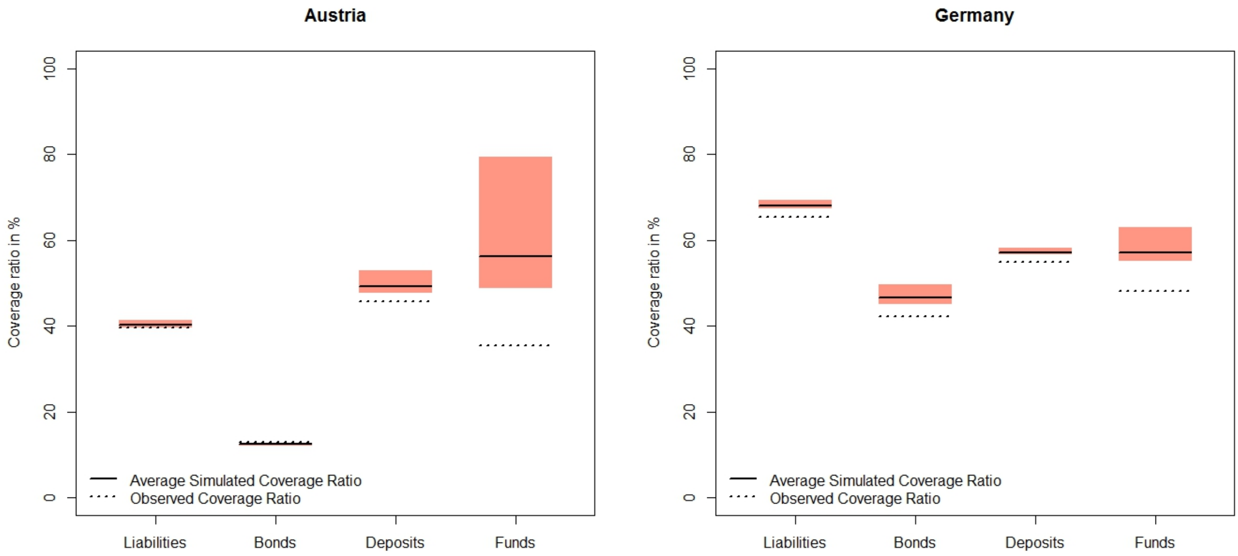
The impact on instrument-specific coverage ratios obviously correlates with the share of assets held by the top tail and the distribution within the tail. The impact on instruments that are less important for the wealthiest of the wealthy (deposits and bonds as well as liabilities) is lower, while the impact on assets which are largely held by the wealthy (funds and equity) is more pronounced.[47]

Indeed, changes are most pronounced for equity. Wealthier households hold much larger shares in equity than the rest of the population and this share increases even more in the top quarter of the tail. The EG-LMM has already documented coverage ratios exceeding 100% when relying on HFCS data only. Most likely, the reason is that there are substantial conceptual and

[46]On average the change is negative, however, a 99% confidence intervals contains the original value and thus no significant change is observed.

[47]For Austria the largest change in coverage ratios is observed for mutual funds. The sensitivity analysis regarding the choice of Pareto threshold (see Appendix C) finds that for mutual funds in Austria the coverage ratio presented here might be overstated.

Figure 9: Coverage ratios.



Notes: The figures plot observed and adjusted coverage ratios. The boxes represent 95% bootstrap confidence intervals.

methodological differences (see discussion in section 6). The Pareto adjustment of the wealth distribution pushes up these numbers even further. Thus, better aligning the HFCS definition of equity with the globally agreed definition in the financial accounts is needed when aiming for distributional indicators.

We also calculate figures for *financial wealth* defined here as the sum of bonds, deposits, equity and mutual funds minus liabilities. Given that parts of equity are recorded as net positions in the financial accounts but spread over all instruments in the HFCS, we would expect better alignment between the financial accounts and the HFCS for this level of aggregation. Adjusting for the missing wealthy leads to over-coverage of financial wealth which is mainly driven by the large increase of equity. This emphasises that differences in net and gross recording are not the only conceptual problems and for instance differences in valuation methods do still play a crucial role.

In general, we find very similar patterns for Austria and Germany, although our methodology leads to larger changes for Austria. This is likely due to the differences in the German and Austrian HFCS in terms of oversampling: Our methodology includes an ex-post adjustment for missing wealthy households. As Germany implements an oversampling strategy, outcomes are more likely to better reflect the upper part of the distribution than in Austria (see Fessler et al., 2016, page 29f for a comparison between the Austrian and German HFCS).

One particular advantage of the simulation approach is that it allows us to calculate empirical confidence intervals. These intervals take into account the Pareto sampling as well as the bootstrap insecurity.[48] Thus, the primary results of our methodology are these *ranges together with point estimates* and not point estimates alone as they hide variability.

Figure 10 shows the share of instrument-specific aggregates held by the tail population, i.e., by

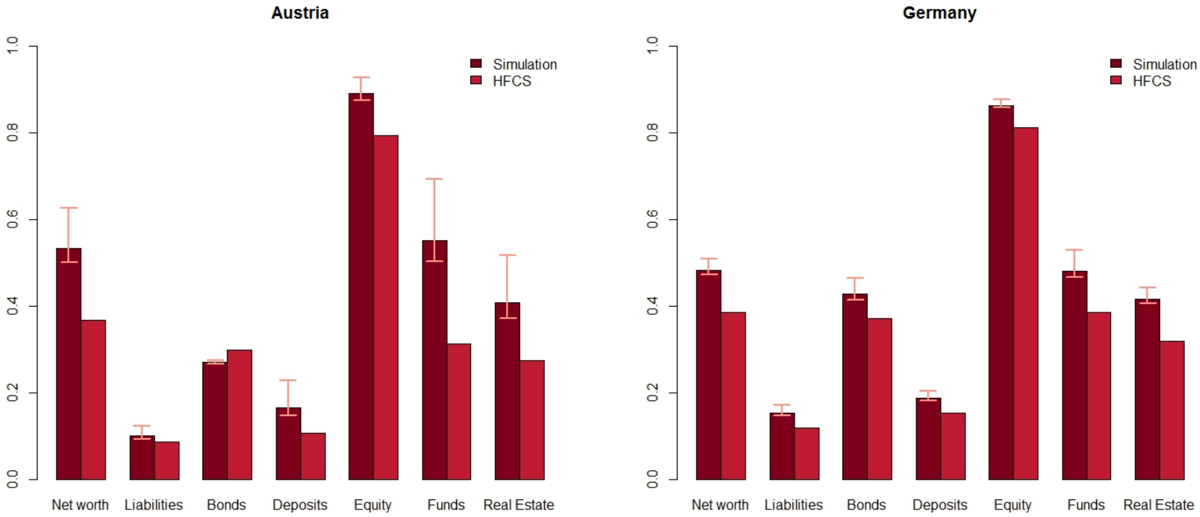
[48]However, the intervals do not take into account any insecurity attached to the choice of methodology to estimate the Pareto shape parameter, or other sources of insecurity resulting from, for instance, multiple imputation and sample design.

Table 8: Adjusted aggregates and coverage ratios.

AT – Austria				
	Observed CR	Adjusted CR	95% confidence interval for CR	Change in CR
Liabilities	39.58	40.24	(39.84; 41.24)	0.66
Bonds	12.90	12.42	(12.34; 12.50)	-0.48
Deposits	45.74	49.37	(47.96; 52.93)	3.63
Equity	155.71	320.41	(259.77; 449.02)	164.70
Mutual Funds	35.55	58.08	(49.27; 78.85)	22.53
Financial Wealth	95.28	180.67	(150.06; 245.33)	85.39
	Observed Aggregate	Adjusted Aggregate	95% confidence interval for Aggregate	Change in Aggregate
Liabilities	66,601	67,714	(67,041; 69,402)	1.67
Bonds	5,272	5,076	(5,042; 5,107)	-3.73
Deposits	98,745	106,592	(103,554; 114,268)	7.95
Equity	194,739	400,729	(324,891; 561,587)	105.78
Mutual Funds	17,070	27,890	(23,575; 38,114)	63.38
Real Estate	686,174	865,514	(793,833; 1,030,398)	26.14
Financial Wealth	249,227	472,572	(392,493; 641,688)	47.26
DE – Germany				
	Observed CR	Adjusted CR	95% confidence interval for CR	Change in CR
Liabilities	65.29	67.99	(67.50; 69.34)	2.70
Bonds	42.30	46.67	(45.30; 49.71)	4.37
Deposits	54.80	57.23	(56.77; 58.24)	2.44
Equity	264.03	363.12	(347.28; 403.72)	99.09
Mutual Funds	48.08	57.33	(55.35; 62.91)	9.25
Financial Wealth	115.50	155.23	(148.87; 171.40)	39.72
	Observed Aggregate	Adjusted Aggregate	95% confidence interval for Aggregate	Change in Aggregate
Liabilities	1,020,920	1,063,155	(1,055,488; 1,084,259)	4.14
Bonds	71,916	79,351	(77,022; 84,520)	10.34
Deposits	1,007,795	1,052,599	(1,044,172; 1,071,207)	4.45
Equity	1,326,078	1,823,723	(1,744,171; 2,027,623)	37.53
Mutual Funds	206,863	246,645	(238,152; 270,661)	19.23
Real Estate	5,872,317	6,859,346	(6,736,002; 7,167,085)	16.81
Financial Wealth	1,591,732	2,139,163	(2,051,511; 2,362,093)	25.59

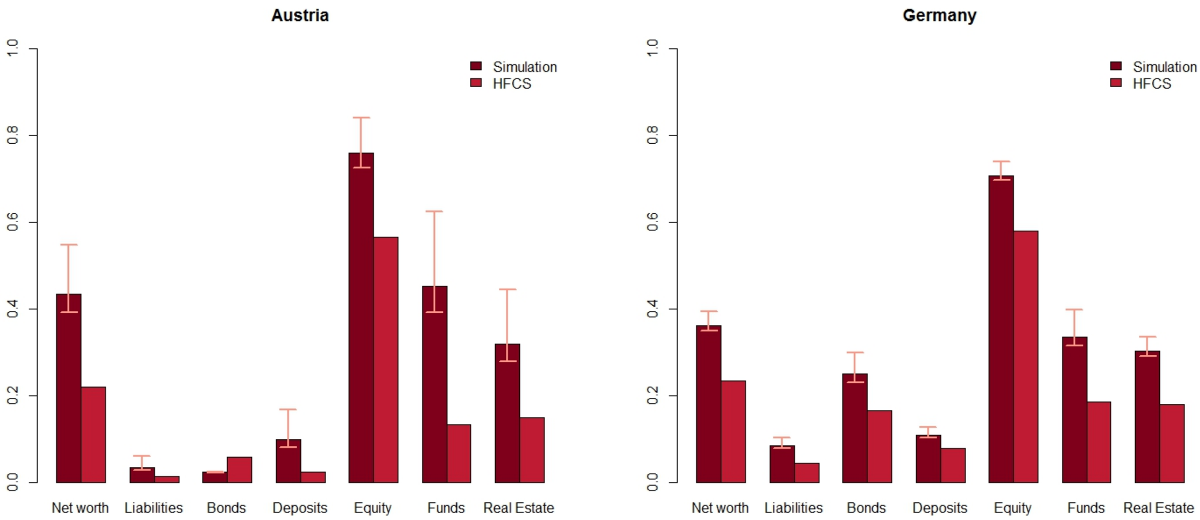
Notes: The table reports observed and adjusted aggregates in mEUR, and coverage ratios (CR) in % per instrument. The change in coverage ratio is given in percentage points, the change in aggregates in %. The 95% confidence interval is obtained from the simulation method whereas point estimates are calculated via the analytical method. Financial wealth is defined as bonds plus deposits plus equity plus mutual funds minus liabilities.

Figure 10: Break down of the wealth of millionaires.



Notes: The figures show the shares of total assets and total liabilities held by the tail population, i.e., by all millionaires. Simulation results also include 95%-confidence intervals.

Figure 11: Break down of the wealth of the top 1%.



Notes: The figures show the shares of total assets and total liabilities held by the top 1% of the population. Simulation results also include 95%-confidence intervals.

all millionaires. Almost all shares[49] increase compared to shares calculated from HFCS data only. Changes are – as expected – least pronounced for liabilities and deposits. We find that millionaires in Austria and Germany hold roughly 90% of total equity.

Similarly, Figure 11 shows the share of instrument-specific aggregates held by the top 1% of the population. Again, shares strongly increase when applying our methodology. Changes are

[49]The negative change for bonds in Austria results from the fact that we observe extremely low shares of net worth invested in bonds at the very top, i.e., in $Q4$.

even more pronounced for equity and real estate as these asset classes are most important for the richest of the rich. While millionaires hold roughly 90% of total equity, the top 1% still hold roughly between 70% and 80% revealing an extremely high degree of wealth inequality for this particular asset class.

Table 9: Comparing methods: Aggregates.

AT – Austria

	Aggregates in mEUR			In … confidence interval			
				95%	99%	95%	99%
	Analytical method	Simulation method	Simple averages	Analytical method		Simple averages	
Liabilities	67,714	67,608	85,292	Yes	Yes	No	No
Bonds	5,076	5,075	10,300	Yes	Yes	No	No
Deposits	106,592	106,100	127,057	Yes	Yes	No	Yes
Equity	400,729	384,467	316,024	Yes	Yes	No	No
Mutual Funds	27,890	26,959	27,155	Yes	Yes	Yes	Yes
Real Estate	865,514	850,486	926,068	Yes	Yes	Yes	Yes

DE – Germany

	Aggregates in mEUR			In … confidence interval			
				95%	99%	95%	99%
	Analytical method	Simulation method	Simple averages	Analytical method		Simple averages	
Liabilities	1,063,155	1,062,378	1,153,936	Yes	Yes	No	No
Bonds	79,351	79,118	95,098	Yes	Yes	No	Yes
Deposits	1,052,599	1,051,348	1,142,704	Yes	Yes	No	No
Equity	1,823,723	1,812,224	1,505,197	Yes	Yes	No	No
Mutual Funds	246,645	245,910	266,968	Yes	Yes	Yes	Yes
Real Estate	6,859,346	6,841,174	7,041,632	Yes	Yes	Yes	Yes

Notes: The table reports aggregates in mEUR obtained from the simulation, analytical and simple averages method. The table also reports whether aggregates fall into the 95%- and 99%-confidence interval obtained from the simulation method.

The strength of the analytical method is to provide accurate point estimates, whereas the simulation method provides additional distributional measures. Of course, the simulation method could also be used to calculate point estimates and – once convergence is reached – there should be only minor differences between aggregates obtained via either approach.

section 4 reports aggregates calculated via the analytical and simulation method. The table also reports results when refraining from stratification and simply taking an average portfolio share for the entire tail population (see Chakraborty et al., 2016). We use the empirical confidence intervals to check whether differences in aggregates are significant.

While differences between aggregates obtained via the analytical and simulation method are almost indistinguishable and insignificant and both the 0.95 and 0.99 level, simple averages lead to fundamentally different conclusions.

Simple averages ignore the correlation between net worth and the portfolio structure. This strongly overestimates liabilities, bonds, deposits, and real estate – instruments that become relatively less important with increasing wealth – and underestimate the most important asset class of the wealthiest of the wealthy: equity. This shows that the more sophisticated approach presented in this paper is absolutely needed.[50]

8 Distributional National Accounts

In this section we give an outlook how our analysis can contribute to enhance the compilation of distributional national accounts (DINA).

The aim of DINA is to compliment national accounts aggregates by distributional indicators. In general, there are many different indicators that would add useful information. Here, we focus on indicators based on the distribution of net worth as we can directly make use of our methodology for this exercise.[51]

We calculate an indicator that splits up financial accounts aggregates by n net worth groups. For instance, $n = 5$ when compiling a spilt-up by net worth quintiles. Such indicators show how much of the total aggregate of a certain instrument is held by for instance the poorest 20% or wealthiest 20% of the population in terms of net worth.

Let y_j denote the FA aggregate for instrument j and

$$x_{\cdot j} = \sum_{i=1}^{n} x_{ij}$$

the HFCS aggregate of instrument j. Thereby, x_{ij} is the HFCS aggregate for instrument j within the net worth group i.

Without adjusting for the missing wealthy in the HFCS, distributional indicators for instrument j and net worth group i are given by

$$d_{ij} = \frac{x_{ij}}{x_{\cdot j}} \cdot y_j.$$

Note that each x_{ij} is scaled up by the inverse instrument-specific coverage ratio. The scaling guarantees that values over all net worth groups add up to the financial accounts total:

$$\sum_{i=1}^{n} d_{ij} = \sum_{i=1}^{n} \frac{x_{ij}}{x_{\cdot j}} \cdot y_j = \frac{y_j}{x_{\cdot j}} \cdot \sum_{i=1}^{n} x_{ij} = \frac{y_j}{x_{\cdot j}} \cdot x_{\cdot j} = y_j.$$

Taking into account the adjustments for the missing wealthy, the HFCS aggregates $x_{\cdot j}$ change whereas the FA aggregates y_j remain constant. To ease notation, assume that by the adjustment, only the highest net worth group is affected, i.e., group n.[52] Let x^*_{nj} denote the adjusted HFCS aggregate for instrument j and net worth group n. The adjusted HFCS total is then given by

$$x^*_{\cdot j} = x^*_{nj} + \sum_{i=1}^{n-1} x_{ij}.$$

[50] Appendix C performs a non-stratified simulation and finds that for highly comparable instruments these confidence intervals do not overlap with confidence intervals obtained from the simulation (except as for mutual funds). This shows that the difference in results are highly significant.

[51] Other indicators may include a split up by income, household structure, or other characteristics. Here we split up aggregates on a household level, i.e., net worth as well as the instruments to be broken down are measured for households, which allows a straightforward comparison. In the case of income or other characteristics that are associated with individuals rather than households, the additional challenge of harmonising the unit of measurement emerges.

[52] For broad indicators such as a split-up by quartiles or quintiles, indeed only the highest group is affected. Changes are, however, quite obvious for more granular split-ups.

Note that in general $x^*_{\cdot j} > x_{\cdot j}$, as our methodology leads to larger amounts held by the wealthy. Distributional indicators are defined analogously via

$$d^*_{ij} = \begin{cases} \frac{x_{ij}}{x^*_{\cdot j}} \cdot y_j, & \text{for } 1 \leq i < n, \\ \frac{x^*_{ij}}{x^*_{\cdot j}} \cdot y_j, & \text{for } i = n. \end{cases}$$

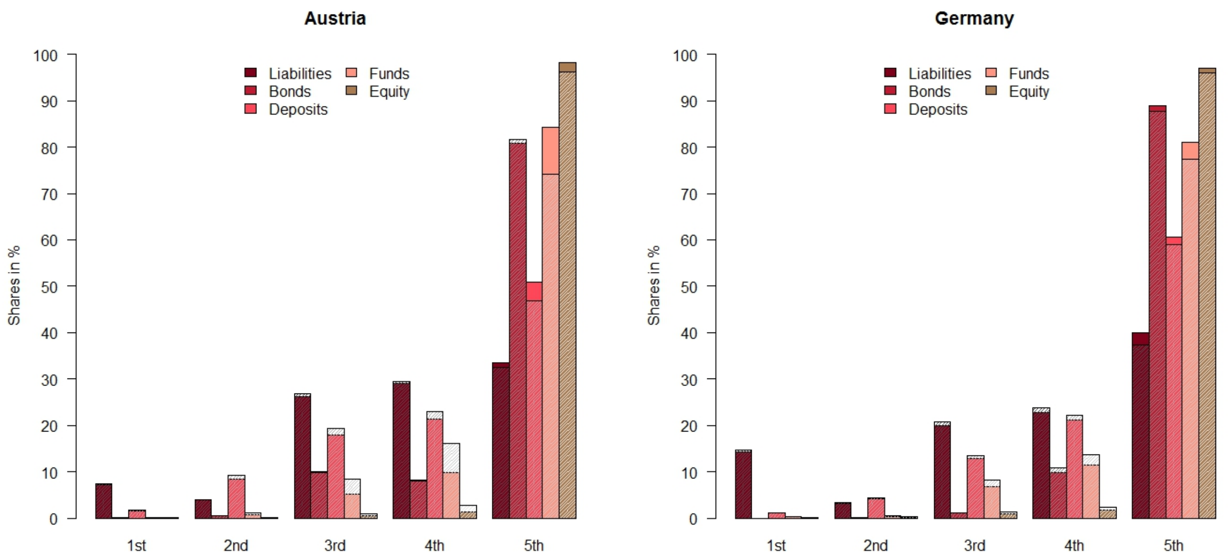
Again, the values over all net worth groups add up to the financial accounts total:

$$\sum_{i=1}^{n} d^*_{ij} = \frac{x^*_{nj}}{x^*_{\cdot j}} y_j + \sum_{i=1}^{n-1} \frac{x_{ij}}{x^*_{\cdot j}} \cdot y_j = \frac{y_j}{x^*_{\cdot j}} \left(x^*_{nj} + \sum_{i=1}^{n-1} x_{ij} \right) = \frac{y_j}{x^*_{\cdot j}} \cdot x^*_{\cdot j} = y_j.$$

Our adjustment has the following effect: For the unadjusted indicators d_{ij} the gap between HFCS and FA aggregates is "filled up" by equally scaling up the respective numbers by the inverse coverage ratio. This means that the "missing" portions are distributed proportionally across wealth groups. Our methodology quantifies the contribution of the missing wealthy to the total gap. Thus, this information is used to allocate this portion of the FA aggregate directly to the top quintile. To fill the remaining gap, aggregates are then again scaled up proportionally.

In theory, if one knew to which net worth quintile to allocate the remaining gap, no scaling would be necessary at all.

Figure 12: Distributional National Accounts.



Notes: The figures show shares of total instrument-specific aggregates held by each net worth quintile (see Table 15). Shaded bars indicate shares without adjustments, i.e., d_{ij}/y_j, whereas full bars indicate adjusted shares, i.e., d^*_{ij}/y_j.

Figure 12 shows results graphically.[53] As we only adjust for the wealthy, it is the top quintile that gets much larger weights for all instruments where our adjustment leads to higher coverage

[53] Table 15 and Table 16 in the appendix report full results as share and Euro amounts, respectively.

ratios (i.e., all instruments except bonds in Austria). At the same time, all other quintiles "loose" (as less Euros are distributed to them due to scaling).[54]

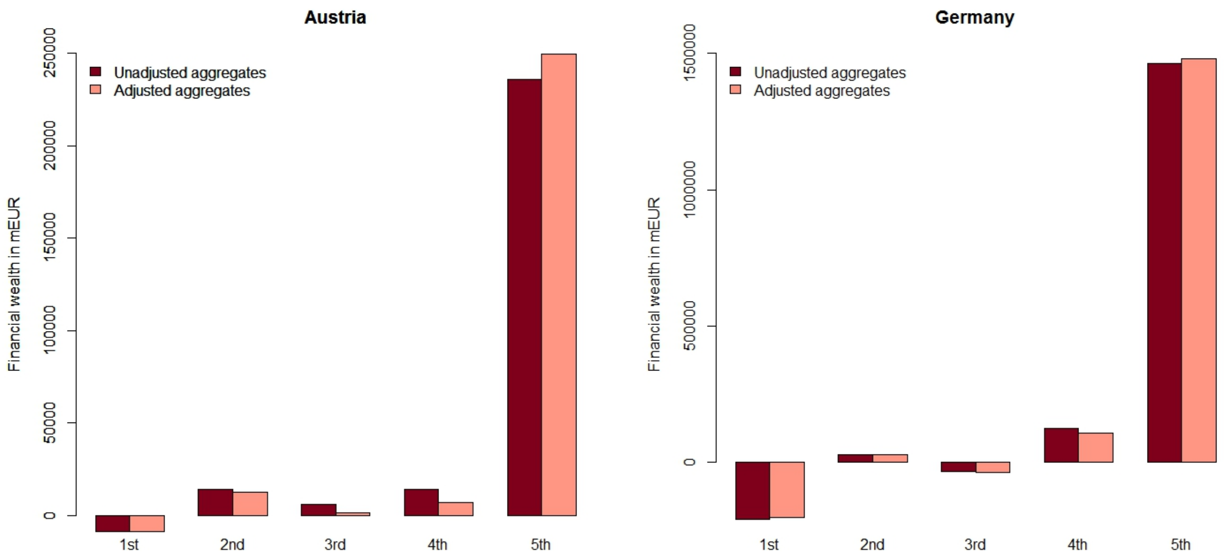
In general, we find very high degrees of instrument-specific inequality which is most dramatic for equity. Inequality is measured to be even larger when applying our corrections. For instance, the share of total mutual funds held by the top quintile is adjusted upwards from 74.2% to 84.2% in Austria and from 77.4% to 81.1% in Germany.

Instrument-specific quintile shares are very similar between Germany and Austria, i.e., we find similar degrees of inequality for both countries *even on instrument-level.* Differences are larger *before* adjusting for the missing wealthy than *after* adjustment. The difference, measured by the sum of squared differences, decreases from

$$\sum_{ij}\left(\frac{d_{ij}(AT)}{y_j(AT)}-\frac{d_{ij}(DE)}{y_j(DE)}\right)^2=0.095 \quad \text{to} \quad \sum_{ij}\left(\frac{d^*_{ij}(AT)}{y_j(AT)}-\frac{d^*_{ij}(DE)}{y_j(DE)}\right)^2=0.084.$$

This finding suggests again, that differences between Austria and Germany in terms of wealth inequality may, at least to a certain extent, be driven by the different treatment of the wealthiest households in the survey: oversampling versus non-oversampling. It, however, also shows that our methodology is likely to overcome parts of this shortcoming.

Figure 13: Distributional figures for Financial Wealth.



Notes: The figures show adjusted and unadjusted total financial wealth (bonds plus deposits plus investment funds plus equity minus liabilities) split up by net worth quintiles. Totals are in mEUR.

We also calculate distributional figures for *total financial wealth*[55] split up by the same net worth quintiles as before, i.e., we summed up financial assets for each net worth quintile (bonds, deposits, mutual funds and equity) and deducted quintile-specific liabilities.

[54]Note that this is a mechanical effect that holds true whenever adjusting for the missing wealthy leads to larger instrument-specific aggregates in the top net worth group. The proof of this statement is given in Appendix D.

[55]The definition of financial wealth sometimes includes pension or insurance entitlements, which we exclude in our analysis due to low conceptual comparability between the HFCS and financial accounts.

A complete set of adjusted DINA figures is provided in Table 10.

The split-up is fully consistent with financial accounts aggregates: Summing over instrument-specific split-ups leads to financial accounts aggregates (as a direct consequence of scaling) and likewise summing over the financial wealth split-up yields total financial wealth as reported by financial accounts.

Table 10: Consistency of Distributional National Accounts.

AT – Austria						
	1st	2nd	3rd	4th	5th	Total
-Liabilities	-12,264	-6,519	-44,254	-48,865	-56,368	-168,269
Bonds	74	237	4,130	3,374	33,053	40,868
Deposits	3,337	18,353	38,591	45,925	109,689	215,895
Equity	42	34	576	1,636	122,781	125,069
Mutual Funds	44	316	2,464	4,746	40,449	48,018
Financial Wealth	-8,767	12,422	1,507	6,816	249,604	261,581

DE – Germany						
	1st	2nd	3rd	4th	5th	Total
-Liabilities	-221,750	-50,051	-311,046	-356,566	-624,171	-1,563,584
Bonds	14	326	1,764	16,764	151,152	170,019
Deposits	20,326	75,095	238,026	389,579	1,116,170	1,839,197
Equity	342	886	4,741	8,706	487,563	502,237
Mutual Funds	1,256	1,561	29,472	49,102	348,842	430,233
Financial Wealth	-199,811	27,816	-37,044	107,584	1,479,556	1,378,102

Notes: The tables report *adjusted* distributional national accounts figures in mEUR consistent with financial accounts totals (as reported in the last column).

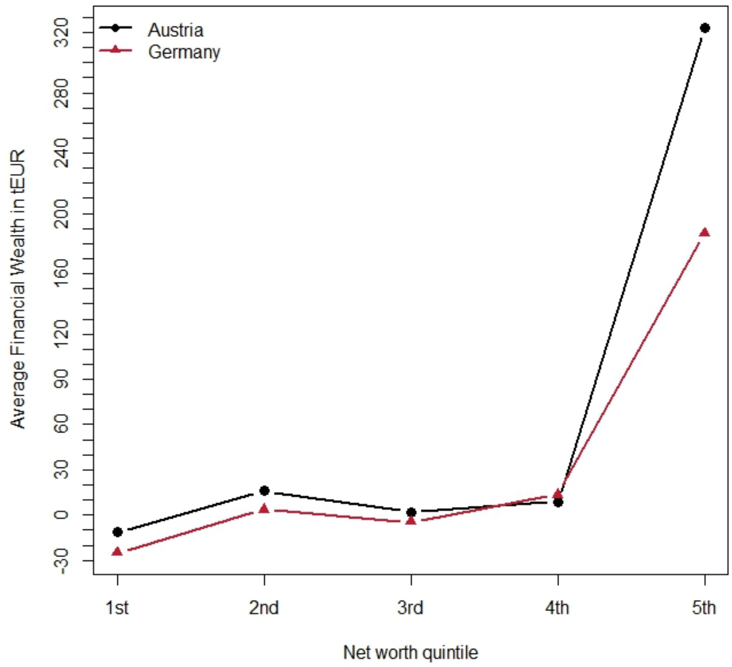
Figure 13 shows the result. In general, the top quintile holds the lion's share of total financial wealth whereas the bottom quintile has negative financial wealth in Austria and Germany. In both countries the 3rd quintile has less financial wealth compared to the 2nd quintile (in Germany the 3rd quintile has even negative financial wealth). This is due to the fact that this part of the population to a large extend substitutes financial wealth with non-financial wealth, i.e., they own more real estate (non-financial assets) that are often financed via mortgages (financial liabilities).

Our adjustment even further increases financial wealth for the top quintile while reducing financial wealth of all other quintiles. In Germany, we find that the top quintile holds 1,480 billion EUR in financial wealth whereas the 4th quintile holds 108 billion EUR only. In Austria, the top quintile holds 250 billion EUR compared to roughly 7 billion EUR held by the 4th quintile.

Figure 14 shows average financial wealth per net worth quintile. In theory, each quintile consists of the same number of households (namely 20% of total households), however, due to survey weights the number of households per quintile varies to some extend.[56] The average financial wealth per household varies strongly over the distribution: In Austria (Germany), a household

[56]For Austria the number of households per quintile ranges between 771,819 and 772,994, and in Germany between 7,920,061 and 7,951,902.

Figure 14: Average Financial Wealth per quintile.



Notes: The figures shows the average financial wealth per quintile in thousand EUR. Numbers are adjusted for the missing wealthy. Financial wealth is defined here as bonds plus deposits plus mutual funds plus equity minus liabilities. It does not include any pension or insurance entitlements.

belonging to the lowest quintile has on average -11,342 EUR (-25,127 EUR) compared to 323,397 EUR (186,789 EUR) for a household belonging to the top quintile.

9 Conclusions

Macroeconomic aggregates are compiled in the system of national accounts which financial accounts form an important part of. Financial accounts are expected to be exhaustive and thus provide an important source of information on the entire economy. On the other side, financial accounts aggregates do not provide any details about the distribution of assets and liabilities within the population. This gap is filled by making use of micro data collected from administrative sources or surveys. This article demonstrates how to use the *Household Finance and Consumption Survey* (HFCS) for such an endeavour.

In an ideal world, without measurement errors, ambiguities in valuation, identical definitions of variables in the survey and financial accounts, a system of national accounts without disturbing balancing effects and a perfect survey sample, we would in expectation get identical aggregates when calculated from either source, i.e., a coverage ratio of 100%. In the real world, coverage ratios are, however, far from perfect.

To achieve the long-term goal of compiling distributional national accounts (DINA) that provide distributional information consistent with aggregates, we need to understand where this gap comes from. This article adds to the literature with this regard as it aims to quantify the impact of the missing rich on the gap between HFCS and financial accounts aggregates, and demonstrates how such a quantification can be used in the compilation of DINA.

We make use of the newly released data of the second wave of the HFCS and analyse the impact of the missing rich in the HFCS on conceptually highly comparable instruments (liabilities, bonds, deposits, and mutual funds).

We perform a case study for Austria and Germany as these countries do not have access to administrative data to strategically oversample the wealthiest households in the HFCS nor to perform plausibility checks and corrections on self-reported survey data. We hence would expect the largest effects for these countries.

Previous findings in the literature suggest that the HFCS underestimates net worth at the top of the distribution. We therefore adjust the distribution by replacing the top tail by a Pareto model. The Pareto model is estimated based on a combined sample of observations from the HFCS and national rich lists.

Additionally, we use portfolio structures observed in the HFCS to break down the Pareto-adjusted wealth at the top tail by instruments. For this purpose, we propose an analytical as well as a simulation approach based on stratified bootstrapping. Whereas the analytical approach is fast and easy-to-implement, the simulation provides additional valuable information on variation and distributional patterns within the tail. Results for aggregates are identical as the simulation method converges to the analytical approach.

We find that coverage ratios consistently increase when explicitly adjusting survey data to better reflect the top tail of the net worth distribution. Changes vary by instrument and are most pronounced for (the conceptually not well comparable item) equity. Although the "missing wealthy" explain up to 20% (but usually less than 10%) of the gap between HFCS and financial accounts aggregates, there still remains a substantial gap which source needs further exploration. Our findings for equity point towards the urgent need to better align the HFCS definition of equity with the globally agreed definition in the financial accounts.

We conduct extensive robustness checks which find that the most crucial assumption in our analysis is the choice of Pareto estimation method. The exact treatment of the rich list is also important. Less influential is the choice of the Pareto threshold and the inclusion of further portfolios to better represent portfolio structures of the wealthiest of the wealthy households. We also find that refraining from stratification yields unreliable results, which emphasise the importance of a more sophisticated approach than just taking simple average as proposed in this article.

We compile distributional account figures and show how our methodology can be used to enhance them. We detect large instrument-specific inequality and find that the top quintile holds the lion's share of financial wealth whereas the bottom quintile has negative financial wealth. In Austria (Germany), a household belonging to the lowest quintile has on average -11,342 EUR (-25,127 EUR) compared to 323,397 EUR (186,789 EUR) for a household belonging to the top quintile. In general, patterns are very similar in Austria and Germany.

As a by-product, we adjust the distribution of net worth. In-line with prior findings in the literature, we find that the HFCS substantially underestimates wealth inequality in Austria and Germany. Our analysis suggests that wealth inequality measured by a household-level Gini coefficient increase from 72% in Austria and from 76% in Germany to roughly 80% in both countries.

In terms of instrument-level inequality, we find that the wealthiest 1% of the population owns 70-80% of total equity and roughly 30% of total real estate assets, but less than 10% of total liabilities. The top wealth quintile holds roughly 98% of total equity in Austria and 97% in Germany, whereas the bottom three wealth quintiles together, i.e., the lowest 60% hold practically nothing in both countries. We also find very high degrees of instrument-specific inequality for mutual funds and bonds.

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Appendix

A The Pareto distribution

A.1 Estimating the shape parameter

A.1.1 (Robust) regression method

The Pareto shape parameter is often estimated using ordinary least squares (OLS), which exploits the linear relationship between the Pareto distribution and the rank of observations: The (shifted) log rank of an observation is a downwards-sloping function of logged wealth with slope parameter ϑ, which is a direct consequence of (1):

$$P(Y > y) = \left(\frac{y}{y_0}\right)^{-\vartheta}$$

and thus

$$\log P(Y > y) = \vartheta \log y_0 - \vartheta \log y.$$

In a finite sample, observations approximately follow a Pareto distribution, if

$$\frac{i}{n} \approx \left(\frac{y_i}{y_0}\right)^{-\vartheta} \qquad \text{for all } i.$$

Thereby, i denotes the rank of the decreasingly ordered observations, y_i net worth of observation i, and n the number of observations. Thus,

$$\log i \approx C - \vartheta \log y_i,$$

with $C = \log(n) + \vartheta \log(y_0)$. The shape parameter ϑ could (in the absence of survey weights) thus be obtained by estimating this equation using OLS.

Vermeulen (2017) adapts this method to take into account survey weights. Additionally, Gabaix and Ibragimov (2011) showed that the estimator for ϑ is unbiased (up to order one) when shifting the rank by 0.5. An estimate for ϑ is then obtained by estimating the equation

$$\log\left((i - 0.5)\frac{\bar{N}_{fi}}{\bar{N}}\right) = C - \vartheta \log(y_i), \tag{3}$$

through a linear model. Thereby, $\bar{N} = \frac{1}{n}\sum_{j=1}^{n} w_j$ is the average weight of all observations and $\bar{N}_{fi} = \frac{1}{i}\sum_{j=i}^{n} w_j$ the average weight of the first i observations.

The major advantage of this approach is that additional observations such as those from a rich list can directly be included in the estimation process. As the response variable is a transformation of the rank of an observation but not its amount, extreme observations from the rich list are less problematic *in the statistical sense.* Still, the different weight structure remains problematic. (Remember that rich list observations are assigned a weight equal to 1 while survey observations may have very large weights. In Germany, the largest weight among all tail observation is 44,189.)

Still, the two types of data (survey and rich list) lead to non-random patterns in residuals. Therefore, we propose a *robust alternative* that can be interpreted as a benchmark method. If, which is the case for our data, differences between the standard regression and robust regression are minimal, confidence in the standard regression is increased.

Quantile regression (which has been developed by Koenker and Bassett Jr, 1978, and is since then widely used in statistics and econometrics) may be used as such a robust alternative to OLS. Quantile regression is per construction less sensitive toward outliers and, in contrast to OLS, does not rely on any distributional assumptions.

Similar as to OLS that estimates the *conditional mean*, quantile regression aims to estimate *conditional quantiles* including the *conditional median.*

For OLS, a linear conditional mean function

$$E[Y|X = x] = x^\top \beta$$

is estimated by solving the *quadratic optimisation problem*

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{n} (y_i - x_i^\top \beta)^2.$$

In a quantile regression setting, the linear conditional median function

$$Q_Y(0.5|X = x) = x^\top \beta(0.5)$$

is estimated by solving the *linear optimisation problem*

$$\hat{\beta}(0.5) = \arg\min_{\beta} \sum_{i=1}^{n} |y_i - x_i^\top \beta|.$$

Thus, quantile regression substitutes a quadratic optimisation problem by a linear optimisation problem. For quantile regression, only the sign of a residual but not its magnitude matters whereas for ordinary least squares the magnitudes are crucial. Hence, quantile regression is robust against outliers whereas OLS is not.

Thus, we use a median quantile regression model to estimate the slope parameter ϑ in equation (3) and call this the *robust regression method*. In contrary, we refer to Vermeulen's method based on OLS as the *regression method*.

A.1.2 Weighted robust estimators

Alfons et al. (2013) recommend the usage of weighted robust estimation procedures for semi-parametric Pareto models based on complex surveys and apply their estimators to income data obtained from the *European Union Statistics on Income and Living Conditions* (EU-SILC). They specifically provide weighted versions of the *integrated squared error* (ISE) estimator and the *partial density component* (PDC) estimator. Details regarding their methodology and further references are found in Alfons et al. (2013).

A drawback of their methodology is that rich list observations are detected as outliers and – as the procedures are designed to be robust against outliers – rich list observations are less influential then needed.

A.2 Pareto quantiles

Quantiles of the Pareto distribution are given by the inverse of the CDF. For a specific quantile level q

$$\begin{aligned} F_Y(y) &= 1 - \left(\frac{y}{y_0}\right)^{-\vartheta} = q \qquad \text{and thus} \\ F_Y^{-1}(q) &= y_0 \cdot (1-q)^{-1/\vartheta} \end{aligned}$$

holds.

Specifically, for the three quartiles we have

$$\begin{aligned} F_Y^{-1}(0.25) &= y_0 \cdot 0.75^{-1/\vartheta}, \\ F_Y^{-1}(0.50) &= y_0 \cdot 0.50^{-1/\vartheta}, \qquad \text{and} \\ F_Y^{-1}(0.75) &= y_0 \cdot 0.25^{-1/\vartheta}. \end{aligned}$$

A.3 Sampling from a Pareto distribution

Sampling from a Pareto distribution is done here by exploiting the inverse probability integral transformation. This theorem states that, given a random variable Y with CDF F_Y, the random variable $Z = F_Y(y)$ follows a uniform distribution $U(0,1)$. Inversely, if $Z \sim U(0,1)$ and $Y \sim F_Y$, then Y and $F_Y^{-1}(Z)$ follow the same distribution.

Thus, it is possible to construct Pareto distributed random numbers from a simple-to-generate sample of uniformly distributed random numbers just by applying the inverse Pareto CDF. Given a uniformly distributed random sample $u_i,\ i = 1, \ldots, n$,

$$y_i = \hat{F}_Y^{-1}(u_i) = \frac{y_0}{(1-u_i)^{1/\hat{\vartheta}}} = \frac{y_0}{u_i^{1/\hat{\vartheta}}}$$

represents a random sample from a Pareto distribution with $\hat{\vartheta}$ and y_0. The last equation is due to symmetry in the interval $(0,1)$.

B Portfolio structures

This section demonstrates how portfolio structures change with net worth and total assets, respectively. We show results based on the HFCS second wave for a series of countries.

The figures are calculated as follows: Observations are increasingly ordered by net worth (total assets) and shares of total assets are calculated for rolling windows. For each window, we calculate the share of total assets (aggregate net worth plus total liabilities) held in different asset classes (y-axis). When calculating totals, we respect sample weights. Additionally, we compute the average net worth (total assets) per window which is assigned to a net worth quantile by empirically evaluating the country's *empirical weighted conditional distribution function of net worth (total assets)* (x-axis). We use a fixed step size of 10 observations and country specific window sizes ranging between 150 observations for Cyprus, 200 for Slovenia, 500 for Austria, Finland and Spain, and 700 for Germany.[57] As we do not include other assets such as vehicles or jewellery, shares do not add up to one.

We find similar structural patterns across countries which are shown in Figure 15 and Figure 16: When analysing shares of total assets broken down by total assets quantiles (i.e., Figure 15), we find two kinds of substitution effects: First, the low end of the total assets distribution hardly holds real estate assets whereas large shares of total assets are held as deposits. When moving up along the distribution, real estate become the single most important asset class. Only at the very top of the distribution, real estate loose importance to assets held in the form of equity – a second substitution effect.

The point where real estate become more important than deposits is quite different across countries. For Austria and Germany – both countries with traditionally very low home-ownership rates[58] – this fix point is roughly at the 45th percentile of the total assets distribution (for Austria it is the 46th percentile which roughly equals EUR 68,300 and for Germany the 43th percentile which roughly equals EUR 49,700). For all other countries, the fix point is reached much earlier.[59] Also the maximum real estate share is noticeably larger for countries other than Austria and Germany.

When looking at shares of total assets broken down by the net worth distribution (i.e., Figure 16), we find very similar patterns for the top tail as liabilities are less important in this part of the distribution. However, at the low end liabilities do play an important role leading to larger real estate shares at the low end of the distribution.

[57]Due to rather large window sizes (which guarantee smooth curves), we lack information for the lowest quantiles.

[58]The 2015 home-ownership rate was 55.7% in Austria and 51.8% in Germany compared to for instance 78.2% in Spain and 72.7% in Finland (*Source:* Eurostat).

[59]We find a strong negative correlation (Pearson correlation coefficient: -0.92) between the fix point and the home-ownership rate among all countries participating in the second HFCS wave.

Figure 15: Shares of total assets for total assets distribution.

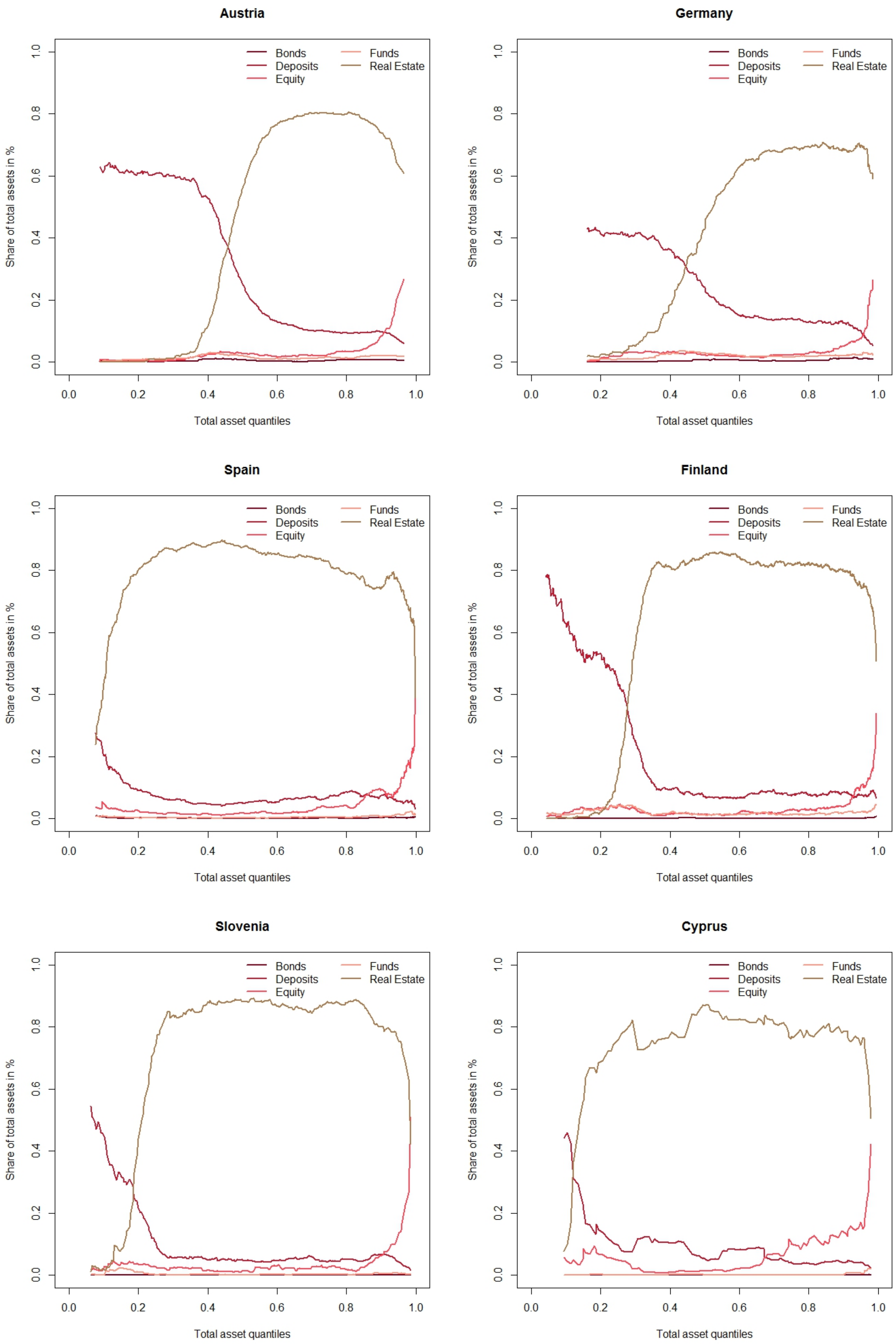


Figure 16: Shares of total assets for net worth distribution.

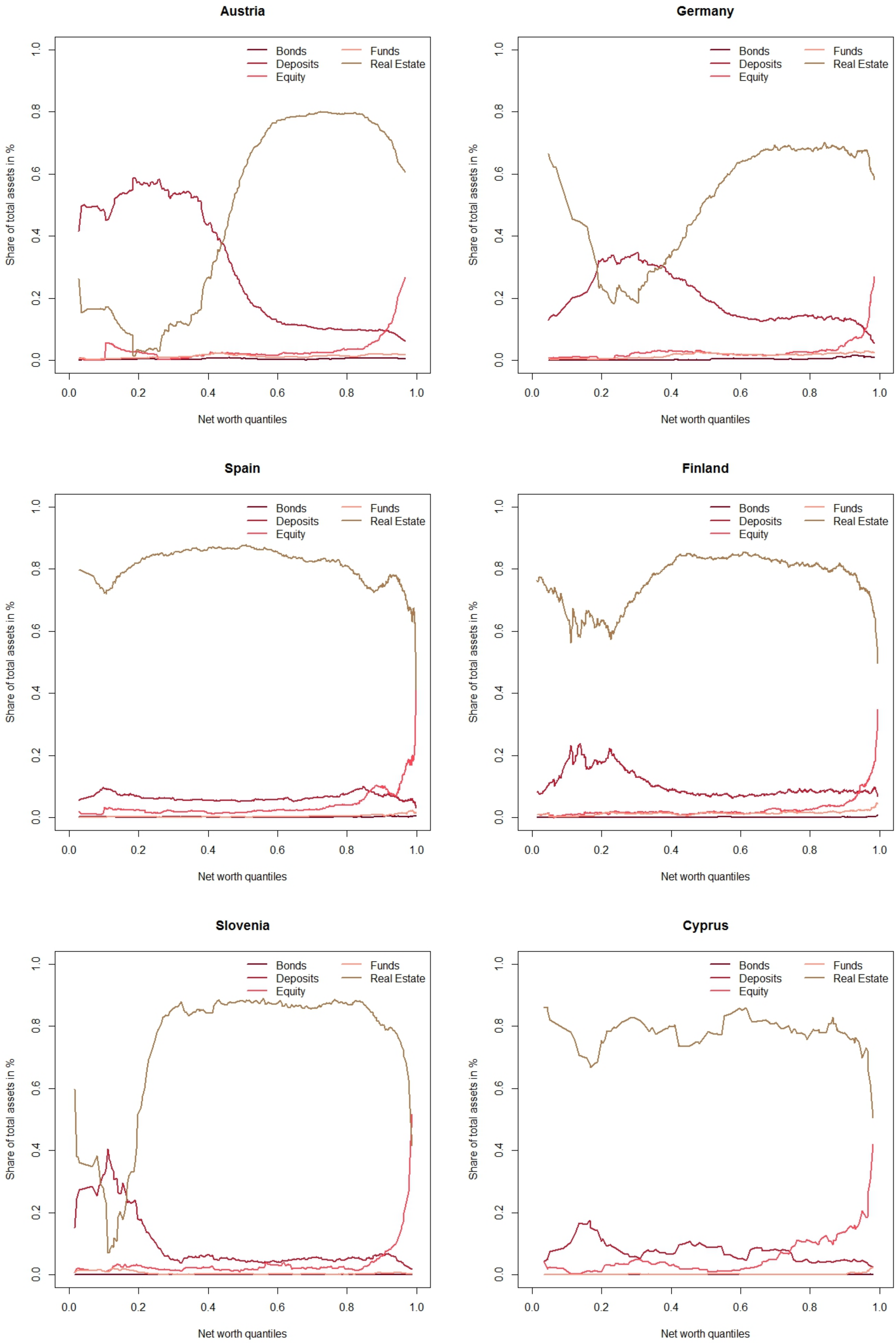
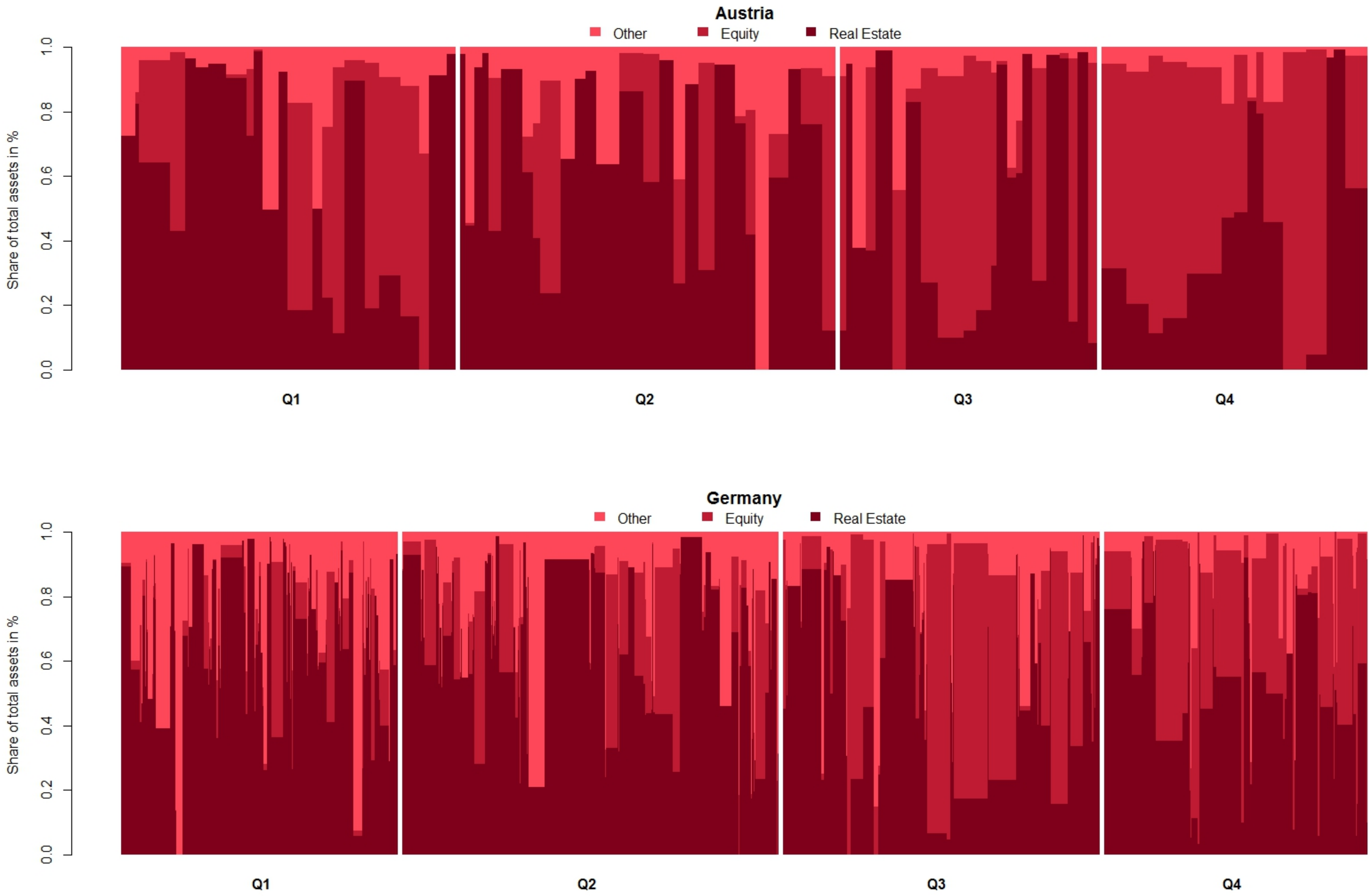


Figure 17: Portfolio allocation of the entire tail.



C Sensitivity analysis

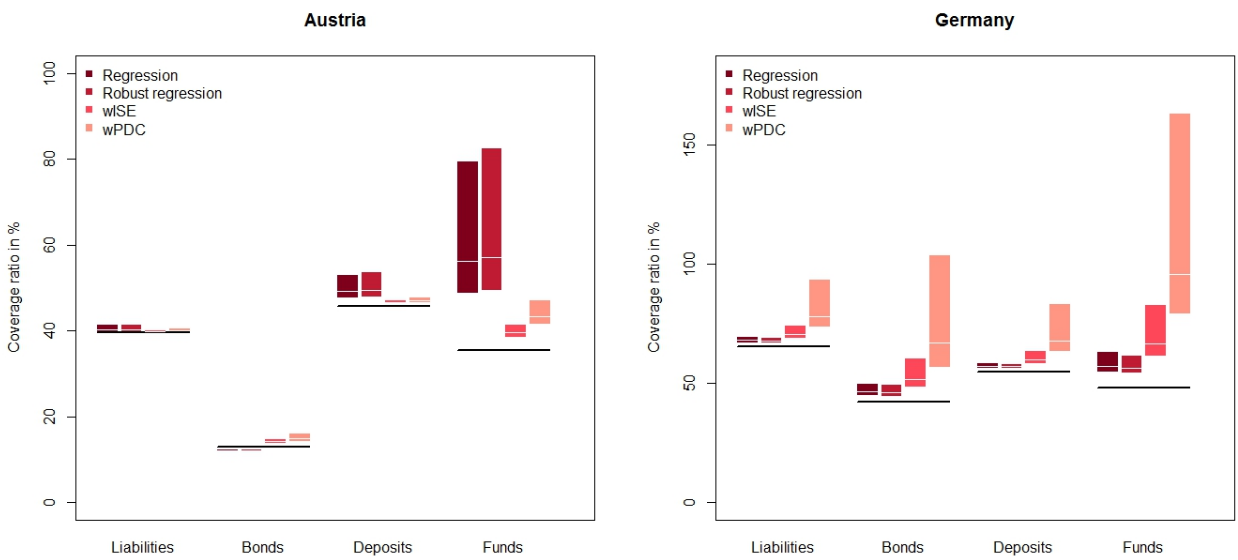
In this section we perform five kinds of sensitivity analyses. We check how much our main results, i.e., shifts in coverage ratios for highly comparable instrument, change when (i) using different methods to estimate the shape parameter of the Pareto distribution ϑ, (ii) using different thresholds from where on the Pareto distribution replaces survey results, (iii) manipulating rich lists used in the Pareto estimation, (iv) refraining from stratifying by net worth, and (v) using additional portfolios from other countries to better reflect the portfolio structure of the very rich.

In general, we find that our methodology achieves a reasonable degree of robustness. In particular, results are extremely robust for liabilities, bonds and deposits, but less so for equity and mutual funds. We expect that results would be even more stable if personal or household level data on wealth and income would be used when performing oversampling. Given the yet scarce knowledge about the distribution of wealth, making available more data sources should in our opinion be of highest priority.

C.1 Choice of Pareto shape parameter estimation method

Estimating the Pareto shape parameter is ambiguous and different methods lead to quite distinct total tail wealth as shown in Table 7. In this section we compare the impact of different methodological choices on *instrument-specific coverage ratios*.

Figure 18: Sensitivity toward the choice of Pareto estimation method.



Notes: The figures show coverage ratios for highly comparable instruments when relying on different Pareto estimation methods. Bars represent 95% confidence intervals, white lines *adjusted* coverage ratios, and black lines *observed* coverage ratios.

We find that the choice of method affects equity the most. This is not surprising as large shares of equity are mainly held by the wealthiest of the wealthy and the likelihood of simulating very wealthy households strongly depends on the exact value of ϑ.

The impact on all other instruments is less pronounced. Figure 18 shows coverage ratios for highly comparable instruments using different estimation methods. Particularly for Germany, the wPDC method leads to large average coverage ratios and wide confidence intervals. This is due to the fact, that this estimation method leads to a much larger and unrealistic total tail wealth compared to all other methods (see Table 6).

Overlapping confidence interval indicate that differences between methods are not significant. For many bilateral comparisons, this indeed holds true: Out of the possible

$$\#\text{instruments} \cdot \begin{pmatrix} \#\text{est. methods} \\ 2 \end{pmatrix} = 4 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 24$$

pairwise combinations, 9 intervals overlap for Austria and 11 intervals overlap for Germany.

C.2 Choice of Pareto threshold

Choosing the threshold parameter y_0 is another major model choice. Whereas the threshold must be large enough to justify the Pareto tail, it should also not be too large to guarantee enough observations that can be used to estimate the Pareto shape parameter. We choose the same threshold (namely one million Euro) for both, Austria and Germany, facilitating comparability. In this section we justify this choice and analyse the effect of varying the Pareto threshold.

There is manifold evidence that the very top of the wealth distribution follows a Pareto law. However, the Pareto model is not suited to describe other parts of the wealth distribution. *Van der Wijk's law* can be used to check, whether the chosen threshold is large enough such that the data already follows a Pareto law.

The Pareto distribution is the only distribution satisfying[60]

$$W(y_0) = \frac{\int_{y_0}^{\infty} y f(y)\ \mathrm{d}y}{\int_{y_0}^{\infty} f(y)\ \mathrm{d}y} = \frac{\vartheta}{\vartheta - 1} \cdot y_0,$$

i.e., the *mean excess function* is proportional to the threshold y_0 (see van der Wijk, 1939; Cowell, 2011b; Dalitz, 2016). Practically, one thus computes the empirical mean excess function over y_0

$$\frac{W(y_0)}{y_0} = \frac{\sum_{y_i > y_0} y_i \cdot w_i}{y_0 \cdot \sum_{y_i > y_0} w_i}$$

for a sequence of thresholds y_0 and checks, whether the chosen threshold lies within the area where $\frac{W(y_0)}{y_0}$ is constant.

Figure 19 shows the results. The van der Wijk statistic is constant from roughly 0.4 million Euro onwards for both countries.

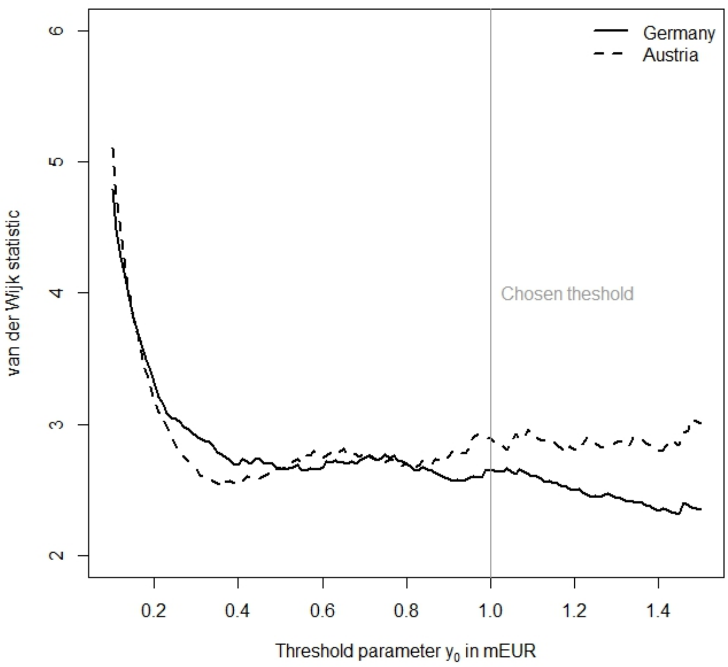
The Kolmogorov-Smirnov test, which calculates the maximum absolute distance between the empirical distribution and a theoretical Pareto distribution, finds minimal deviation at $y_0 = 0.56$ mEUR for Austria and $y_0 = 0.41$ mEUR for Germany.

Thus, these two theoretical criteria suggest rather low thresholds. Additionally, we calculate instrument-specific coverage ratios for various thresholds ranging between 0.4 and 2.1 million Euro and thus also for the widely used thresholds[61] of 0.5, 1 and 2 million Euro.

[60] Since the support of the Pareto distribution is (y_0, ∞), we have $\int_{y_0}^{\infty} f(y)\ \mathrm{d}y = \int_{-\infty}^{\infty} f(y)\ \mathrm{d}y = 1$ and $\int_{y_0}^{\infty} y f(y)\ \mathrm{d}y = \int_{-\infty}^{\infty} y f(y)\ \mathrm{d}y = E(Y)$. Using (2) thus yields $W(y_0) = \frac{\vartheta}{\vartheta-1} \cdot y_0$. So, the Pareto distribution satisfies van der Wijk's law. It is also easy to show that among all continuous distributions the Pareto distribution is in fact the only one satisfying this relationship (see Cowell, 2011b, Appendix A.3 for a proof).

[61] See Vermeulen (2017), Chakraborty et al. (2016), and Bach et al. (2015).

Figure 19: Choice of Pareto threshold parameter.



Notes: The figure shows the van der Wijk statistic for Austria and Germany calculated from HFCS data.

Figure 20 shows the results. We find that (except for mutual funds in Austria) coverage ratios are fairly stable for thresholds above one million EUR. For equity, coverage ratios are in general more volatile and become unreliable for thresholds above two million Euro.

We therefore conclude that the minimal threshold suggested by the van der Wijk law and the Kolmogorov-Smirnov test are too low and thus go for a larger threshold achieving satisfying results according to all our analyses: one million Euro. A larger threshold is also beneficial in the sense that we do not want to substitute too large parts of the survey by a parametric model.

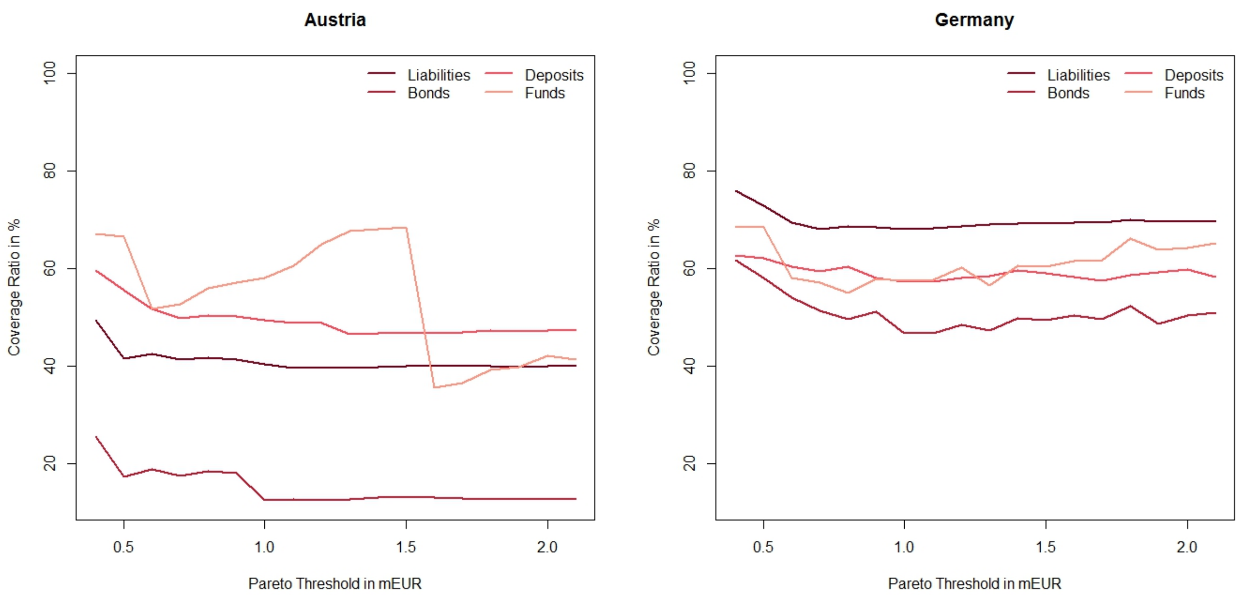
C.3 Rich list observations

As the HFCS lacks observations from the very top, estimating the wealth distribution from this data only is likely to underestimate the top tail. Thus, we enrich the HFCS data with observations from rich lists and use this combined data set when estimating the Pareto shape parameter.

Rich lists provide us with observations from the far right of the distribution. However, as rich lists cover only 100 (AT) and 500 (DE) of the riches households, there is still a large gap between the largest observation in the HFCS and the smallest observation on the rich list.

In this section we therefore test, how results change when manipulating the rich list. We manipulate rich lists in four ways: the exact number of observations on the list, the influence of the largest observation, the treatment of observations as single households or rather family clans consisting of several households, and the exclusion of households holding assets mainly in the form of foundations.

Figure 20: Sensitivity toward the choice of Pareto threshold.



Notes: The figures show coverage ratios for highly comparable instruments when using different Pareto thresholds. Thresholds range between 0.4 and 2.1 million Euro with a step size of 100,000 Euro.

We estimate the Pareto shape parameter ϑ using the regression method when relying on *(i)* the full rich list, *(ii)* the top 50% of all rich list observations, *(iii)* the top 25% of all rich list observations, and *(iv)* the top 10% of all rich list observations. We also estimate the shape parameter when excluding the wealthiest household from the rich list *(v)* as this is, in particular for Austria, an extreme observation[62] with possibly large leverage.

As a number of observations on the rich list may refer to family clans that probably consist of more than one single household, we test the effect of splitting the amount into several "households." We perform this exercise once by splitting *all* observations into *(vi)* two or *(vii)* four households, respectively. Alternatively, we report the results for a *(viii) random split*: We split each observation into u households, whereas $u \sim U(1, 4)$ follows a discrete uniform distribution on the set $\{1, 2, 3, 4\}$. We repeat this 10,000 times and report the average result for $\hat{\vartheta}$ and the average length of the simulated rich list.[63]

For Austria, the rich list also contains information on the major source/type of wealth: foundations, inherited wealth (without further classification), and listed or unlisted shares. Thus, we measure the effect of *ix* excluding observations from the rich list that, according to the Trend list, "hold assets" only in the form of foundations (see footnote 39 for a discussion about foundations in Austria).

Results are reported in Table 11. We find that in general the exact number of observations from rich lists does not matter a lot as seen in rows *(ii)–(iv)*. Even a strikingly small number, such

[62] In Germany, the richest family (Quandt/Klatten: BMW) owns 31 billion Euro (according to the Manager Magazine) whereas the second wealthiest family (Albrecht/Heister: Aldi Süd) "only" owns 18.3 billion, i.e., roughly 60% of the Quandt/Klatten fortune. In Austria, Porsche/Piëch (Porsche, VW) are estimated to own 65 billion Euro (according to Trend), whereas the second-ranked Dietrich Mateschitz (Red Bull) owns with 7.6 billion Euro only roughly 12% of the Porsche/Piëch fortune.

[63] Bach et al. (2015) split up German rich list observations based on information they collected through "thorough internet research." They claim that there is not enough information for lower ranked rich list observations. For this part of the list, they hence assume that each observation refers to four households.

Table 11: Sensitivity toward the number of rich list observations.

		AT – Austria			DE – Germany		
		Obs.	$\hat{\vartheta}$	Tail wealth	Obs.	$\hat{\vartheta}$	Tail wealth
(i)	full rich list	100	1.194	778,285	500	1.338	4,899,939
(ii)	top 50% of rich list	50	1.174	854,177	250	1.317	5,144,155
(iii)	top 25% of rich list	25	1.176	844,238	125	1.306	5,277,994
(iv)	top 10% of rich list	10	1.179	832,291	50	1.318	5,125,318
(v)	excl. top observation	99	1.211	726,232	499	1.341	4,866,363
(vi)	split into 2 households	200	1.205	744,288	1,000	1.368	4,600,772
(vii)	split into 4 households	400	1.221	699,804	2,000	1.387	4,432,071
(viii)	random split $U(1,4)$	250	1.214	717,052	1,250	1.376	4,528,258
(ix)	excl. foundations	77	1.234	666,607	–	–	–

Notes: The table reports estimation results based on the regression method when manipulating the rich list. Namely, we use the top 50%, top 25%, and top 10% of the rich list, analyse the effect of leaving out the top observation, "foundation" observations, and the effect of splitting total amounts on the rich list into two, four and a random integer between 1 and 4 households. Tail wealth is given in mEUR and calculated according to formula (2).

as 10% of the original list, leads to a very similar total tail wealth. Results are less sensitive for Germany as there are in general many more observations.

Leaving out the richest family clan – see row *(v)* – in Austria has a stronger effect than in Germany. This is not surprising given the extreme difference in net worth between the wealthiest and second wealthiest family in Austria (see footnote 62).

Splitting amounts on the rich list into several households – see rows *(vi)*–*(viii)* – to account for the fact that at least some observations refer to family clans consisting of several households, does have a stronger effect than leaving out parts of the rich list. Whereas shortening the rich list increases tail wealth, splitting observations leads to a decrease.

Results for leaving out households which hold their assets in the form of foundations are presented in row *(ix)*. As this information is not available for Germany, results are only reported for Austria. Total tail wealth is reduced as many top (including *the* top) observation is removed.

Table 12 reports the effects of such manipulations on instrument-specific coverage ratios. We report coverage ratios for the maximum and minimum estimated shape parameter according to Table 11. While changes in total tail wealth seem to be quite remarkable, the impact on coverage ratios are negligible except as for mutual funds and equity.

C.4 Stratification

Our methodology relies on stratification taking into account the observation that portfolio structures change when moving along the net worth distribution. Although we believe that stratification increases reliability in our methodology, we here show results when neglecting this correlation.

This means that in case of the simulation approach for each simulated net worth, we re-sample an arbitrary portfolio from the full list of observed portfolios in the tail population. As a consequence, the correlation between net worth and portfolio structure is *not preserved.* For instance, a household with high simulated net worth is *not* more likely to hold greater shares

Table 12: Effects of manipulating rich lists on coverage ratios.

	Austria				Germany		
	(ii)	*(i)*	*(vii)*	*(ix)*	*(iii)*	*(i)*	*(vii)*
Liabilities	40.37	40.24	39.80	39.75	68.58	67.99	67.05
Bonds	12.45	12.42	13.22	13.16	47.39	46.67	48.18
Deposits	49.99	49.37	49.07	48.81	57.85	57.23	57.21
Equity	351.47	320.41	315.32	300.71	389.79	363.12	331.97
Mutual Funds	61.38	58.08	51.99	50.70	59.35	57.33	54.28

Notes: The table reports effects of manipulating rich lists on coverage ratios. Results are in % and derived using the analytical approach. We report results for the largest and smallest change in estimated Pareto shape parameter (see Table 11) and compare it to the standard method denoted by *(i)*. For Austria, we additionally report changes when excluding foundations.

of equity than a household a simulated net worth close to one million Euro.

Likewise for the analytical approach, we do not calculate stratum-specific portfolio shares but rely on a single average portfolio structure for the entire tail.

Table 13 reports results:[64] Aggregates for instruments with decreasing importance when moving to the top end of the distribution (liabilities, deposits, bonds, and real estate) increase. In contrast, aggregates for equity, which is most relevant for the very top of the tail, decreases. As there is no clear correlation between the importance of mutual funds and net worth, this instrument is least affected when refraining from stratification.

These findings underpin the importance of stratification. In the future, when more waves of the HFCS become available, portfolio structures from previous waves could be added to increase the precision.[65] This might be particularly helpful for small countries like Austria. (Also see the discussion in the next section.)

C.5 Representativenss of portfolio structures

As argued in subsection 3.2 surveys that do not oversample the wealthiest households might not appropriately reflect portfolio structures of the very rich. Thus, adding additional portfolios to the top stratum may be desirable to increase reliability. This section shows how HFCS data obtained from other countries could be used with this regard.

When analysing figures Figure 15 and Figure 16, we find for Austria and Germany that at the very top the average share of total assets or net worth held as equity is much lower than the average share held as real estate. This is not plausible for the wealthiest of the wealthy. In other countries, including heavily oversampling Spain and regionally oversampling Slovenia, equity becomes more important than real estate at the very top. Thus, particularly for Austria and Germany borrowing portfolio structures from other countries to better reflect the very top of the distribution seems important. The plausible assumption behind this approach is that

[64]See also subsection 7.2.

[65]Some countries including Germany already have or plan to have a panel component. When using portfolio structures from previous waves, only the latest panel observation should be used to avoid bias. However, in the future panel observations can be used to assess the stability of portfolio structures over time. Currently, we draw portfolios respecting their weights, i.e., a portfolio from an observation representing many households is more likely to be (and therefore is more often) drawn than one from an observation with a lower weight. When merging portfolios from several waves, the weight structure is not obvious any more.

Table 13: Effects of stratification.

AT – Austria				
	Adjusted Aggregate (stratified)	Adjusted Aggregate (not stratified)	95% confidence interval for CR (stratified)	95% confidence interval for CR (not stratified)
Liabilities	67,714	85,292	(39.84; 41.24)	(47.00; 58.79)
Bonds	5,076	10,300	(12.34; 12.50)	(20.39; 36.06)
Deposits	106,592	127,057	(47.96; 52.93)	(54.92; 67.48)
Equity	400,729	316,024	(259.77; 449.02)	(206.34; 364.17)
Mutual Funds	27,890	27,155	(49.27; 78.85)	(48.00; 78.61)
Real Estate	865,514	926,068	–	–

DE – Germany				
	Adjusted Aggregate (stratified)	Adjusted Aggregate (not stratified)	95% confidence interval for CR (stratified)	95% confidence interval for CR (not stratified)
Liabilities	1,063,155	1,153,936	(67.50; 69.26)	(72.82; 76.07)
Bonds	79,351	95,098	(45.30; 49.89)	(53.43; 62.51)
Deposits	1,052,599	1,142,704	(56.78; 58.35)	(61.23; 64.48)
Equity	1,823,723	1,505,197	(347.13; 407.37)	(286.79; 330.59)
Mutual Funds	246,645	266,968	(55.36; 62.19)	(59.62; 68.66)
Real Estate	6,859,346	7,041,632	–	–

Notes: The table reports adjusted aggregates (in mEUR) and 95% confidence intervals for coverage ratios (in %). Results rely on adjusting the tail with and without stratification, respectively. The 95% confidence interval are obtained from the simulation approach whereas aggregates rely on the analytical approach.

the wealthiest of the wealthy across Europe have similar investment habits.

We thus supplement the top stratum, $Q4$, with portfolio shares from other HFCS countries representing the top of the top. More specifically, for a given level q (here we use $q \in [0.99; 0.9999]$) we calculate the quantile y_q (i.e., the threshold in terms of net worth such that q% of the observations are lower and $(1-q)$% are larger), and filter all observations from the entire HFCS (except the particular country we analyse) that have a net worth of a least y_q. This set of "foreign" observations is denoted by F. Finally, we add F's portfolio shares to the top stratum $Q4$.

When adding additional portfolios, we need to adjust the weights in $Q4$. Therefore, we calculate for each observation in F its *relative weight* by dividing its *frequency weight* by the total number of households in the respective country and denote them by ω_i^F. We scale these relative weights so that they add up to one within F:

$$\frac{\omega_i^F}{\sum_{i \in F} \omega_i^F}.$$

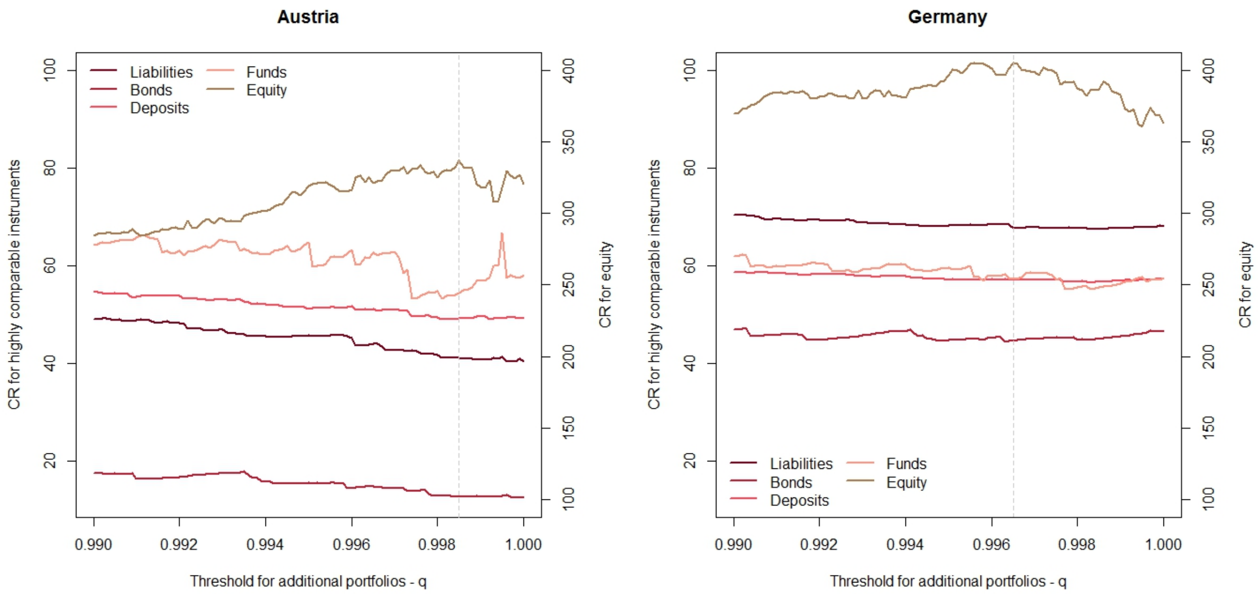
Denoting the total population by N, i.e., the sum over all weights for the country of analysis, *frequency weights* are thus obtained by

$$w_i^F = (1-q) \cdot N \cdot \frac{\omega_i^F}{\sum_{i \in F} \omega_i^F}.$$

Portfolio shares together with frequency weights in F are then added to $Q4$. To guarantee that the population total in $Q4$ remains unchanged, we divide *all weights*, i.e., those originally from $Q4$, w_i^{Q4}, and those added from foreign countries, w_i^F, by the sum of total frequency weights and multiply them by the number of households in $Q4$:

$$0.25 \cdot N \cdot \frac{w_i^{F \cup Q4}}{\sum_{i \in F \cup Q4} w_i^{F \cup Q4}}.$$

Figure 21: Representativeness of top portfolio shares.



Notes: The figures show coverage ratios for various thresholds q.

Figure 21 shows resulting coverage ratios for various thresholds q. In general we find that coverage ratios for highly comparable instruments are not heavily affected by the inclusion of additional portfolios. Comparing the two countries, changes are larger for Austria and in particular for mutual funds in Austria.

A priori, it is not clear which threshold q is most appropriate. As we expect wealthiest households to hold most of their assets as equity, we choose the threshold that achieves the largest aggregates for equity which is indicated as dashed line in Figure 21:

$$q^* = \underset{q \in [0.99;0.9999]}{\arg\max} \; A_{\text{equity}}(q).$$

Results for q^* are reported in Table 14. We find that instrument-specific coverage ratios are only marginally affected by adding additional portfolios and qualitative conclusions remain unchanged. Due to our maximisation approach, changes are naturally largest for equity.

For Austria, we find $q^* = 0.9985$ and $y_{q^*}^{AT} = 13,237,765$ EUR, and for Germany $q^* = 0.9965$ and $y_{q^*}^{DE} = 5,965,796$ EUR.[66] Across the entire HFCS, 187 (and thereof 186 non-Austrian) observations exceed $y_{q^*}^{AT}$ and 553 (and thereof 528 non-German) exceed $y_{q^*}^{DE}$. Most of these

[66] Quantiles are calculated from the semi-parametric Pareto model. We therefore combine non-tail HFCS observations with a simulated sample from the estimated Pareto model representing the adjusted tail. We select the sample used here out of 100 draws according to a Kolmogorov-Smirnov test.

Table 14: Usage of additional foreign portfolio structure at the very top.

	AT – Austria		**DE – Germany**	
	No extra obs.	Extra obs. $q^* = 0.9985$	No extra obs.	Extra obs. $q^* = 0.9965$
Liabilities	40.24	41.04	67.99	67.63
Bonds	12.42	12.70	46.67	44.63
Deposits	49.37	49.08	57.23	56.99
Equity	320.41	336.70	363.12	405.22
Mutual Funds	58.08	54.44	57.33	57.17

Notes: Coverage ratios are obtained by the analytical method. *No extra obs.* refers to the standard method used in this article. *Extra obs.* makes use of additional portfolios of the wealthiest households observed in the entire HFCS, whereas we only supplement the q^*th quantile of the net worth distribution with extra observations. Coverage ratios are in %.

foreign observations come from Spain (112 in AT / 247 in DE), followed by France (52 in AT / 209 in DE). Spain and France are countries performing advanced oversampling strategies based on individual wealth data and achieved the highest effective oversampling rate of the top 5%[67] among all countries participating in the second wave of the HFCS (see HFCN, 2016a, Table 4.7).

The added portfolios are mainly of type "equity," i.e., the most important asset class is equity (in 75.3% of all cases in Austria and 57.0% in Germany). Thus, these additional portfolios fulfil our expectations about portfolio structures of the wealthiest of the wealthy.

D Derivations

D.1 Analytical method

This section derives stratum-specific average wealth needed in section 4. Therefore, we use the inverse of the Pareto CDF F^{-1} (see equation (1) and Appendix A). Quantiles in general are calculated as described in Appendix A and denoted by $F^{-1}(p)$.

Strata are defined as quartiles of the tail, i.e., $Q = [F^{-1}(p_1); F^{-1}(p_2))$, with $\{p_1, p_2\} \in \{\{0, 0.25\}, \{0.25, 0.5\}, \{0.5, 0.75\}, \{0.75, 1\}\}$. Note that $F^{-1}(0) = y_0$ and $F^{-1}(1) = \infty$.

Stratum-specific average wealth is given by

$$\begin{aligned} W(Q) = E(Y|Y \in Q) &= \frac{E(Y \cdot \mathbb{I}_Q(Y))}{P(Y \in Q)} \\ &= \frac{1}{p_2 - p_1} \int y \mathbb{I}_Q(y) \, \mathrm{d}F(y) = \frac{1}{p_2 - p_1} \int_{F^{-1}(p_1)}^{F^{-1}(p_2)} y \, \mathrm{d}F(y), \end{aligned}$$

whereas

$$\mathbb{I}_Q(Y) = \begin{cases} 1, & Y \in Q, \\ 0, & Y \notin Q \end{cases}$$

denotes the indicator function.

[67]See subsection 6.1 for a definition of the effective oversampling rate.

Substituting $q = F(y)$ yields $\mathrm{d}F(y) = \mathrm{d}F(F^{-1}(q)) = \mathrm{d}q$. The thresholds $F^{-1}(p_1)$ and $F^{-1}(p_2)$ change to p_1 and p_2, respectively, yielding

$$\begin{aligned} W(Q) &= \frac{1}{p_2 - p_1} \int_{p_1}^{p_2} F^{-1}(q)\ \mathrm{d}q = \frac{1}{p_2 - p_1} \int_{p_1}^{p_2} y_0 \cdot (1-q)^{-1/\vartheta}\ \mathrm{d}q \\ &= \frac{\vartheta y_0}{(1-\vartheta)\cdot(p_2 - p_1)} \left[(1-p_2)^{1-1/\vartheta} - (1-p_1)^{1-1/\vartheta}\right]. \end{aligned}$$

D.2 Properties of DINA

This section proves a property about DINA as defined in section 8: If instrument-specific aggregates belonging to the top net worth group are increased (by for instance adjusting for the missing wealthy), then distributional national account figures d_{ij} are strictly adjusted downwards for all groups except the top and strictly adjusted upwards for the top group.

As instrument-specific aggregates belonging to the top net worth group are increased, total instrument-specific aggregates rise, i.e., $x^*_{\cdot j} > x_{\cdot j}$. This inequality implies

$$d_{ij} > d^*_{ij} \quad \text{for } 1 \le i < n, \tag{4}$$

i.e., for all but the highest net worth group, and all instruments j. Thus,

$$\begin{aligned} \sum_{i=1}^{n} d_{ij} &= \frac{x_{nj}}{x_{\cdot j}} y_j + \sum_{i=1}^{n-1} d_{ij} \overset{(4)}{>} \frac{x_{nj}}{x_{\cdot j}} y_j + \sum_{i=1}^{n-1} d^*_{ij} \\ &= \frac{x_{nj}}{x_{\cdot j}} y_j + \sum_{i=1}^{n} d^*_{ij} - \frac{x^*_{nj}}{x^*_{\cdot j}} y_j = \frac{x_{nj}}{x_{\cdot j}} y_j + \sum_{i=1}^{n} d_{ij} - \frac{x^*_{nj}}{x^*_{\cdot j}} y_j, \end{aligned}$$

whereas the last equality is a direct consequence of scaling, i.e., $\sum_{i=1}^{n} d_{ij} = y_j = \sum_{i=1}^{n} d^*_{ij}$. This implies

$$d_{nj} = \frac{x_{nj}}{x_{\cdot j}} y_j < \frac{x^*_{nj}}{x^*_{\cdot j}} y_j = d^*_{nj},$$

which finalises the proof.

E Tables

The following tables reports the full set of adjusted and unadjusted DINA results for Austria and Germany as described in section 8.

Table 15: Distributional National Accounts – Shares.

AT – Austria		Liabilities		Bonds		Deposits		Equity		Mutual Funds	
Quintiles		d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j
1st	$(-\infty; 6.9]$	7.4	7.3	0.2	0.2	1.7	1.5	0.1	0.0	0.1	0.1
2nd	$(6.9; 35.4]$	3.9	3.9	0.6	0.6	9.2	8.5	0.1	0.0	1.1	0.7
3rd	$(35.4; 162.0]$	26.7	26.3	9.7	10.1	19.3	17.9	0.9	0.5	8.4	5.1
4th	$(162.0; 365.5]$	29.5	29.0	7.9	8.3	23.0	21.3	2.7	1.3	16.1	9.9
5th	$(365.5; \infty)$	32.4	33.5	81.6	80.9	46.9	50.8	96.2	98.2	74.2	84.2

DE – Germany		Liabilities		Bonds		Deposits		Equity		Mutual Funds	
Quintiles		d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j	d_{ij}/y_j	d^*_{ij}/y_j
1st	$(-\infty; 2.5]$	14.8	14.2	0.0	0.0	1.2	1.1	0.1	0.1	0.3	0.3
2nd	$(2.5; 27.4]$	3.3	3.2	0.2	0.2	4.3	4.1	0.2	0.2	0.4	0.4
3rd	$(27.4; 113.0]$	20.7	19.9	1.1	1.0	13.5	12.9	1.3	0.9	8.2	6.9
4th	$(113.0; 274.0]$	23.7	22.8	10.9	9.9	22.1	21.2	2.4	1.7	13.6	11.4
5th	$(274.0; \infty)$	37.4	39.9	87.8	88.9	58.9	60.7	96.0	97.1	77.4	81.1

Notes: The table reports shares of total instrument-specific aggregates held by each net worth quintile. The column "quintile" reports the thresholds of net worth in tEUR. Shares are in %. Quintile thresholds are calculated using HFCS data (as our adjustment only affects the top 1-2%, these thresholds do not change in the course of our adjustment). Adjusted ratios use results from the analytical method.

Table 16: Distributional National Accounts – Euro Amounts.

AT – Austria

Quintiles		Liabilities		Bonds		Deposits		Equity		Mutual Funds	
		d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}
1st	$(-\infty; 6.9]$	12,469	12,264	71	74	3,602	3,337	86	42	71	44
2nd	$(6.9; 35.4]$	6,628	6,519	229	237	19,811	18,353	70	34	517	316
3rd	$(35.4; 162.0]$	44,994	44,254	3,976	4,130	41,658	38,591	1,186	576	4,025	2,464
4th	$(162.0; 365.5]$	49,682	48,865	3,248	3,374	49,574	45,925	3,367	1,636	7,754	4,746
5th	$(365.5; \infty)$	54,497	56,368	33,345	33,053	101,250	109,689	120,360	122,781	35,651	40,449

DE – Germany

Quintiles		Liabilities		Bonds		Deposits		Equity		Mutual Funds	
		d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}	d_{ij}	d^*_{ij}
1st	$(-\infty; 2.5]$	230,923	221,750	16	14	21,230	20,326	470	342	1,498	1,256
2nd	$(2.5; 27.4]$	52,122	50,051	359	326	78,434	75,095	1,218	886	1,861	1,561
3rd	$(27.4; 113.0]$	323,914	311,046	1,946	1,764	248,608	238,026	6,520	4,741	35,140	29,472
4th	$(113.0; 274.0]$	371,317	356,566	18,497	16,764	406,898	389,579	11,973	8,706	58,545	49,102
5th	$(274.0; \infty)$	585,308	624,171	149,201	151,152	1,084,027	1,116,170	482,056	487,563	333,190	348,842

Notes: The table reports amounts of total instrument-specific aggregates held by each net worth quintile. The column "quntiles" reports the thresholds of net worth in tEUR. Amounts are in mEUR. Quintile thresholds are calculated using the definition of net worth in the HFCS (as our adjustment only affects the top 1-2%, quintile thresholds do not change). Adjusted aggregates use results from the analytical method.