World Inequality Lab – Working Paper N° 2021/03

A Micro Perspective on r > g

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January 2021

A MICRO PERSPECTIVE ON $r > g$*

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February 2, 2021

Abstract

For which segments of the net wealth distribution are returns to wealth higher than growth rates of income? Although Piketty's $r > g$ is conceived as a tool to highlight distributional concerns, it does not account for covariation between real rates of return and position in the net wealth distribution. By exploiting large-scale administrative data on estimated gross and net personal wealth in Norway from 2010 to 2018, this paper establishes the first micro-level analysis of the difference between the real return on wealth and the real growth rate of total pre-tax income across the entire net wealth distribution. We show that, for the top half of the distribution, the aggregate $R - G$ underestimates its micro counterpart $r - g$, whilst the opposite happens for the bottom half, indicating that the micro $r - g$ qualifies as a more precise measure to thoroughly analyze the dynamics of wealth inequality. We show as well that around 44% of the variation in $r - g$ when moving up from the bottom to the top decile of the net wealth distribution is associated with scale dependence, whilst the residual variation can be attributed to persistent heterogeneity.

Keywords: *Wealth inequality; Income inequality; Norway.*
JEL Classification: *D30, D31, D33.*

*We are grateful to Y. Berman, E.E. Bø, M. J. Dávila-Fernández, C. Martinéz-Toledano, B. Milanovic, M. Morgan, S. Morelli, T. Piketty, M. Ranaldi, A. Roventini for comments and suggestions. The technology to access the data remotely, Microdata.no, was developed in a collaboration between the Norwegian Centre for Research Data (NSD) and Statistics Norway as part of the infrastructure project RAIRD, funded by the Research Council of Norway. The views expressed herein are those of the authors and do not necessarily reflect those of our institutions. All errors are our own.

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1 Introduction

The publication of "*Capital In The Twenty-First Century*" (Piketty, 2014) has sparked a surge in interest in the study of wealth inequality and the relation between the rate of return on capital and the growth rate of income. The bottom line in Piketty (2014) and Piketty and Zucman (2014) is that if the rate of return on wealth overcomes that on income ($r > g$), wealth-rich individuals, the so called *rentiers*, would accumulate wealth faster than individuals typically holding low or negative values of wealth and mainly relying on income, thus fostering wealth disparities in the longer run. The necessary assumptions for this prediction to hold and the relation to economic theory have been analyzed by Hiraguchi (2019); Jones (2015), Mankiw (2015) and Stiglitz (2016). The author himself returns to the debate in Piketty (2015a), clarifying that he does not consider *"$r > g$ as the only or even the primary tool [...] for forecasting the path of inequality in the twenty-first century. Institutional changes and political shocks [...] played a major role in the past, and it will probably be the same in the future"*.

In our view, a thorough understanding of $r > g$, its predictive power, relevance, and eventual limitations both in the short and longer run, hinges crucially on the variety of analyses carried out upon it. Several studies have recently attempted at decomposing the rate of return on wealth, in order to allow heterogeneity of returns across the wealth distribution. Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) use granular asset price data and find that the relation $r > g$ is a constant feature of their data in peacetime, for every country and time period under analysis.[1] Focusing on r, Fagereng, Guiso, Malacrino, and Pistaferri (2020) exploit the high quality of Norwegian individual-level data on wealth holdings to document the persistent heterogeneity of real rates of return on net worth across the distribution, even within asset classes. Furthermore, they show that scale dependence matters since rates of return on net worth are positively correlated with individuals' position in the wealth distribution. Bach, Calvet, and Sodini (2020) use Swedish data and confirm that the expected return on (household) net worth is strongly persistent and increasing with net wealth holdings.

Proceeding along these lines, we intend to fill a gap in the literature by providing the first micro-level empirical assessment of the difference between r and g across the net wealth distribution, in relation to its aggregate $R - G$ counterpart. By exploiting large-scale administrative data on personal wealth in Norway from 2010 to 2018, we show that the aggregate $R - G$ (with an average of $1,1\%$ throughout the period) underestimates its micro counterpart $r - g$ for the top half of the wealth distribution, whilst the opposite happens for the bottom half. This implies as well that the micro $r - g$ predicts a higher level of income and wealth inequality, in comparison to $R - G$. This result is illustrated through a simple simulation exercise in the paper. In other words, although formally the macro $R - G$ can be expressed in terms of its micro counterpart $r - g$ through a difference between two weighted averages, our empirical evidence indicates that the distribution of

[1]For Norway in the period $1980 - 2015$, they estimate on average that the real return on wealth is 6.55% higher than the real growth of GDP.

$r - g$ provides insights on the dynamics of inequality of income and wealth that do not arise by exclusively focusing on mean variables.

We analyze as well whether our evidence on the micro $r - g$ can be explained only by persistent heterogeneity across the net wealth distribution, or if we can attribute some of its variation to scale dependence. Results show that around 44% of the variation in $r - g$ when moving up from the bottom to the top decile of the net wealth distribution is associated with scale dependence, implying that the scale of wealth can indeed be inserted among the determinants of the micro $r - g$. Finally, we decompose personal wealth into its main components (housing and financial), to show that the share of financial wealth is positively correlated with real rates of return, whilst the opposite is true for housing wealth.

The paper is structured as follows. Section 2 presents the data and outlines our definitions of personal income and wealth. Section 3 presents the main results, followed by the discussion section 4, before section 5 concludes the paper.

2 Data and descriptive statistics

Our analysis is based on Norwegian administrative tax records on income and wealth.[2] Norwegian administrative tax records represent a particularly reliable source of information since most components of income and wealth are reported by third parties, such as banks and employers, mitigating the risk of measurement errors and under-reporting deriving from self-reported income and wealth in surveys.

Our baseline sample consists of the entire population of residents in Norway with age 20 years and above (although our results are not affected by considering a younger sample), in between 2010 and 2018. For each resident individual i, the following definitions of personal wealth, capital income and total fiscal income (all pre-tax) are considered. All variables are measured at the last day of the year and are at the level of individuals, not of households.

Gross wealth [$gw_{i,t}$]: taxable individual gross wealth, including estimated market values of real and financial capital. Real capital includes estimated market value of the primary dwelling, secondary dwellings, land and buildings related to business activity. Financial capital includes cash, domestic deposits, foreign deposits, government and corporate bonds, bond funds and money market funds, shares in stock funds, other taxable capital abroad and outstanding claims and receivables.

Private debt [$d_{i,t}$]: private debt to Norwegian and foreign creditors (consumer debt, student debt and long-term debt), including debt related to shares in real estate companies.

Capital income [$k_{i,t}$]: taxable property income includes share dividends, interest income on bank deposits and on domestic and foreign assets, interest on loans to companies, realized capital gains, imputed rents and capital gains on housing. From this we subtract realized capital losses and interest expenditure. We compute imputed rents as a constant

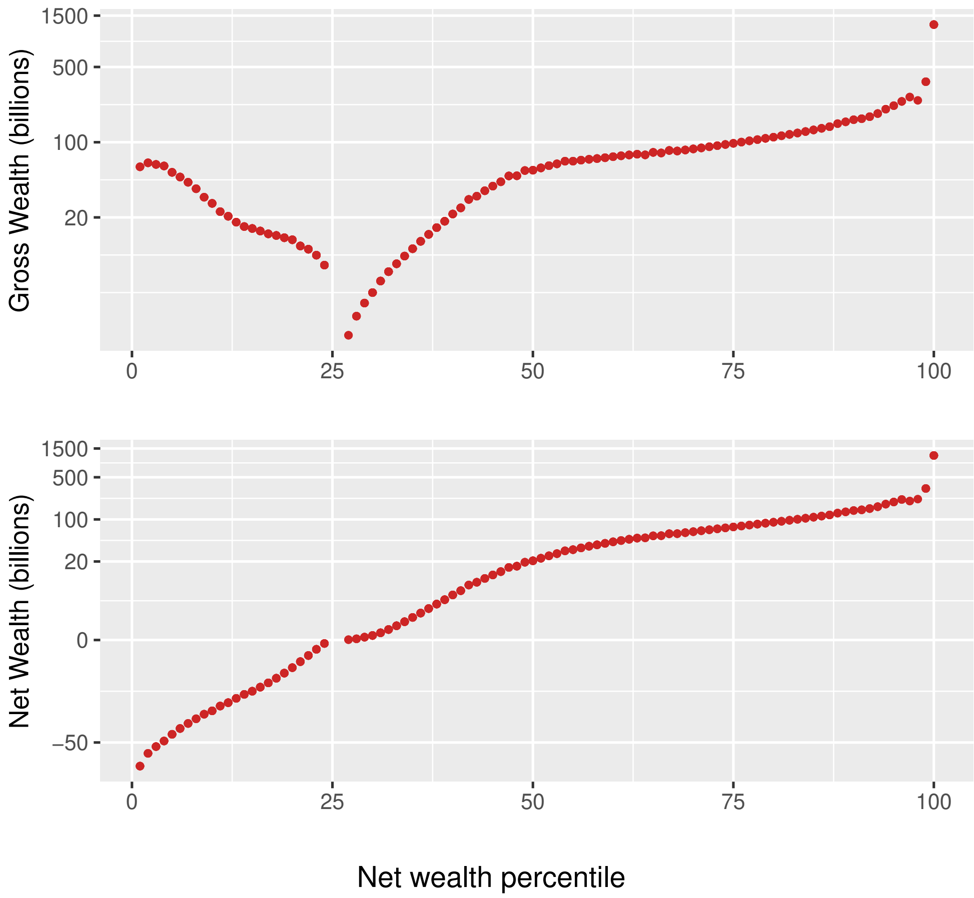
[2]Data are retrieved from microdata.no, an online portal administered by Statistics Norway. For replication purposes, the full dataset and Stata .do files used to obtain the results will be made publicly available.

fraction of the percentile estimated value of housing wealth by employing a nominal interest rate of 3%, as done in Bø (2020). We follow Fagereng et al. (2020) and compute capital gains on housing as the yearly difference on housing wealth of the previous year.

Total fiscal income [$y_{i,t}$]: pre-tax fiscal income includes employee income and net income from self-employment[3], taxable and tax-free transfers and capital income.

Table 4 in Appendix A shows summary descriptive statistics describing our full sample (before any trimming), which varies from around 3.67 million individuals in 2010 to 4.12 millions in 2018, and it sums up to 35.09 millions throughout the period. All variables are subsequently adjusted for inflation based on CPI and expressed from here onward in real terms.[4]

Figure 1: Gross and Net Wealth, 2011 − 2018.



Note: this figure shows the total percentile gross and net wealth (billions Norwegian kroner, constant prices, 2015 CPI, pseudo-log scale), ranked across the net wealth distribution, and pooled across the years 2011 − 2018. The bottom part of the gross wealth distribution appears to be decreasing since individuals are ranked according to their net wealth holdings.

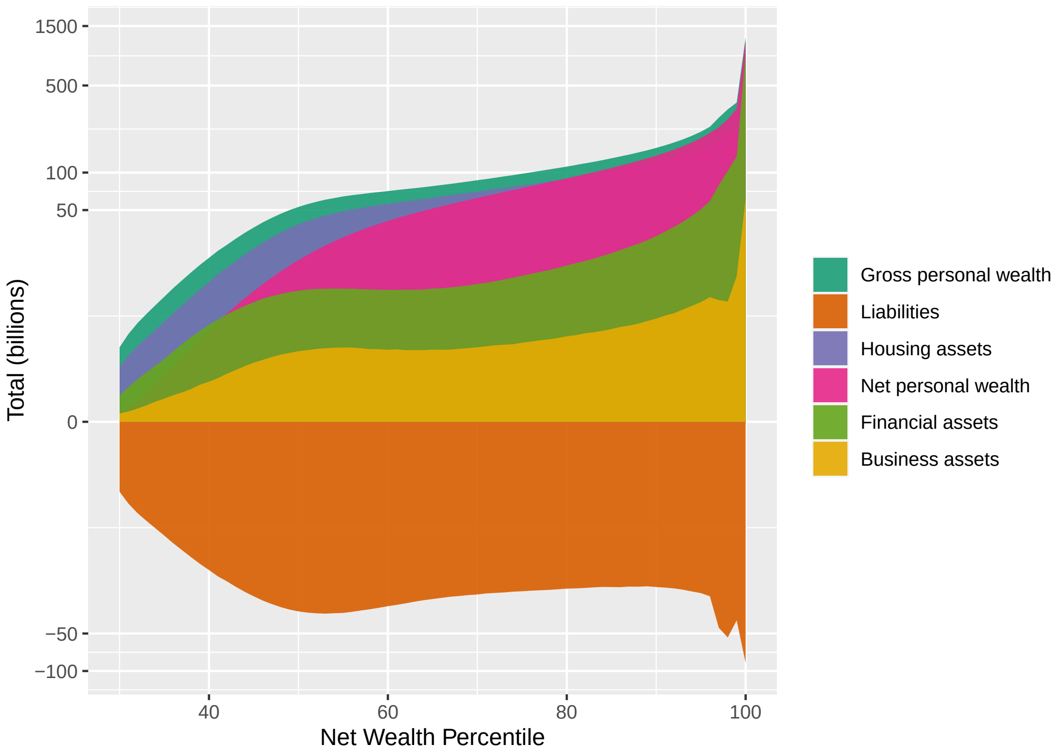
[3] Net self-employment income is the sum of self-employment income in agriculture, forestry and fishing and self-employment income from other industries received during the calendar year, less any losses. It also includes sickness benefits paid to the self-employed.

[4] For each variable, our totals in each year match those from the household sector wealth statistics provided by Statistics Norway (Table 10315 - Property account for households 2010 − 2018). In Microdata.no, the top 1% is winsorized due to privacy concerns. We hence impute the difference between our winsorized series and Statistics Norway totals to the top 1%.

Figure 1 plots the average gross (gw_t) and net ($w_t = gw_t - d_t$) wealth total percentile values (billions Norwegian kroner, constant prices, 2015 CPI), ranked across the net wealth distribution. Due to indebtedness in the lower deciles (mostly long-term debt), net wealth turns positive only around the 25th percentile. The gross wealth distribution exhibits a Gini coefficient of 0.54, whilst the Gini for the net wealth distribution rises to 0.61.

Figure 2 shows the different components of personal wealth in Norway across the net wealth distribution. Notably, the wealthy own higher shares of financial and business assets with respect to the rest of the distribution, while liabilities are substantially high throughout the distribution, highlighting the high level of households' indebtedness in the Norwegian economy.

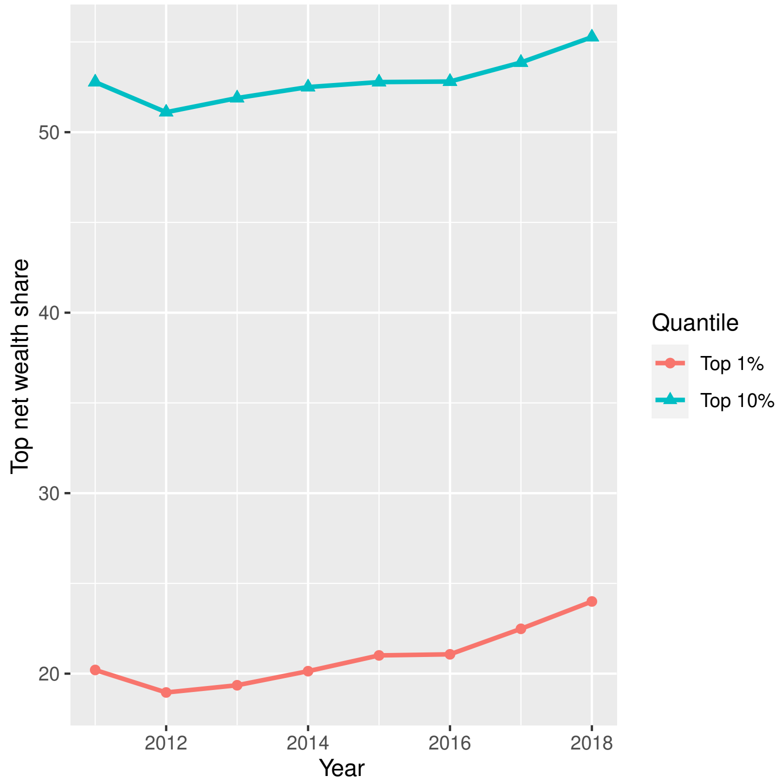
Figure 2: The composition of wealth, $2011 - 2018$.



Note: The composition of wealth in Norway across the net wealth distribution. Averages (pooled across the years, $2011 - 2018$) per percentile, nominal values. The y-axis is represented in a pseudo-log scale.

Proceeding with measures of wealth concentration, Figure 3 plots the shares for the top 10% and top 1% of the net wealth distribution from 2011 to 2018. The top 10% receives a slightly increasing share, in between 50% and 55% of the total net wealth in our sample. The same is true for the top 1%, increasing its share from around 20% to 24% in the final year. A top 1% share of slightly above 20% is in line with previous estimates of top wealth shares in Norway, documented in Epland and Kirkeberg (2012).

Figure 3: Shares of net wealth for the top 1% and top 10%, $2011-2018$.

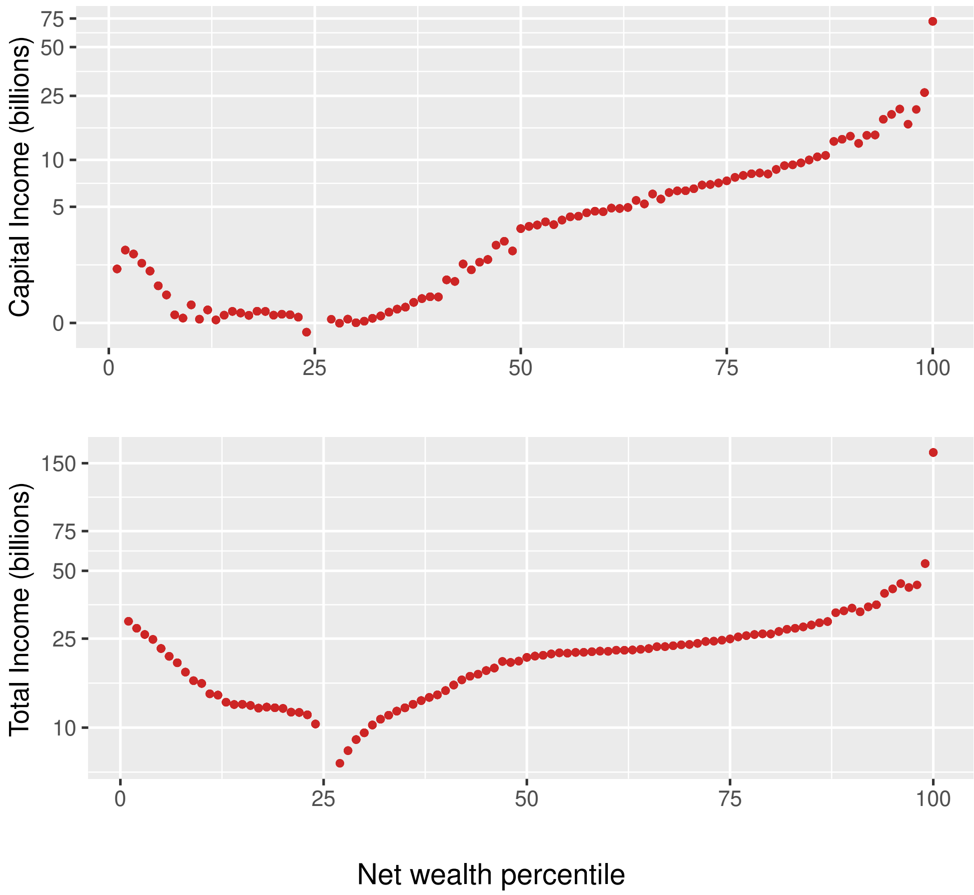


Note: this figure plots the $2011-2018$ time series for the shares for the top 10% and top 1% of the net wealth distribution.

Finally, Figure 4 plots the series of capital (k_t) and total income (y_t) across the net wealth distribution. The Gini coefficient for the distribution of pre-tax capital incomes exhibits a level of 0.62, whilst it drops to 0.27 for the series of pre-tax total income (this value is in line with estimations of the Gini coefficient of total income for Norway by Statistics Norway, which lies in between 0.237 in 2011 and 0.251 in 2018).[5] The discrepancy between our estimates of the Gini and that of Statistics Norway might be due to our capital income definition which is net of interest expenditure, and that includes imputed rents and unrealized capital gains on housing.

[5]Statistics Norway Table 09114: Measures of income dispersion. Household equivalent income (EU-scale) between persons (M) (UD) 2004 - 2018.

Figure 4: Capital and Total Income, 2011 − 2018.



Note: this figure plots the distributions of capital and total income (billions Norwegian kroner, constant prices, 2015 CPI, pseudo-log scale), ranked across the net wealth distribution and pooled across the years 2011 − 2018.

3 Results

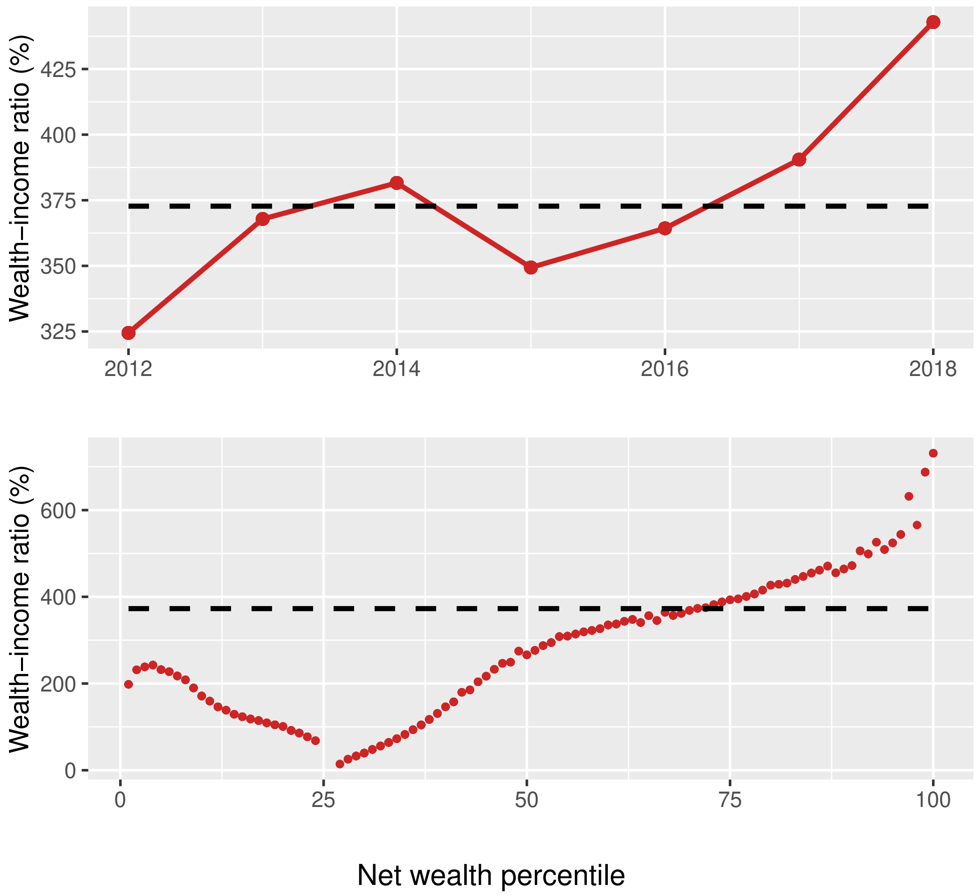
3.1 Wealth-income ratios

In the unified framework developed by Piketty and Zucman (2014) and Alvaredo et al. (2016), national wealth is the sum of public and private wealth, where private wealth consists of net wealth of private households (personal wealth), and of non-profit institutions serving households (NPISH). In this work, we focus purely on personal wealth, hence abstracting from net wealth of NPISH and from public wealth. This choice allows, however, a more precise mapping between the aggregate and the micro variables. We start by presenting our estimates of the household sector's aggregate wealth-income ratio β for each t:

$$\beta_t = \frac{GW_t}{Y_t} = \frac{\sum_{p=1}^{P} gw_{p,t}}{\sum_{p=1}^{P} y_{p,t}}, \tag{1}$$

where gw_p and y_p are respectively the percentile sums of individual-level real gross wealth and total income, and with $P = 100$ since we focus on percentiles. In addition, we derive the micro βs for the pooled sample given by $\beta_p = \frac{gw_p}{y_p}$, showing how the wealth-income ratio evolves across the net wealth distribution.

Figure 5: Wealth-income ratio: aggregate and by percentile



Note: the upper part of this figure shows the aggregate wealth-income ratio across the years $2012 - 2018$, whilst the lower part shows the micro wealth-income ratio across the distribution of net wealth. The average is 372% and is marked by a horizontal dashed line both in the upper and lower parts of the figure.

The upper part of Figure 5 shows how the aggregate wealth-income ratio in our sample evolves over the period considered. The average throughout the period is 372% (marked by a horizontal dashed line both in the upper and lower parts of the Figure). Our aggregate wealth-income ratio grows non-monotonically from 325% in 2012 to slightly below 450% in 2018.[6] The lower part of Figure 5 shows instead how the wealth-income ratio varies across the distribution of net wealth throughout the period. For the top 30%, the wealth-income ratio lies above the aggregate average of 372%, whilst the opposite is true for the bottom 70%. The top 1% of the net wealth distribution exhibits a wealth-income ratio slightly above 700%, indicating a high degree of heterogeneity across the distribution and

[6]For a comparison, Fagereng, Holm, Moll, and Natvik (2019) show that in between 2012 and 2015 Norway's aggregate wealth-to-income ratio (they label this series as *"No saving by holding"*) ranged from around 450% to around 480%.

especially at the very top.

3.2 The aggregate R and G

We define the aggregate real rates of return R as the yearly ratio between end-of-period total capital income at time t (net of interest expenditure, the cost of capital) and end-of-period total gross wealth at $t-1$. Following Fagereng et al. (2020), we express rates of return as a share of gross wealth to avoid negative values for individuals with negative net wealth, and to avoid measurement errors attributing infinite returns to individuals with very low values of net wealth.

$$R_t = \frac{K_t}{GW_{t-1}} = \frac{\sum_{p=1}^{P} k_{p,t}}{\sum_{p=1}^{P} gw_{p,t-1}}. \tag{2}$$

Our estimate of the rate of return in Norway, pooled across the years $2012 - 2018$, exhibits an average of 4.83%. Further, we define the aggregate growth rate G of total fiscal income as follows:

$$G_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\sum_{p=1}^{P} y_{p,t} - \sum_{p=1}^{P} y_{p,t-1}}{\sum_{p=1}^{P} y_{p,t-1}}. \tag{3}$$

Our estimate of the growth rate G of total fiscal income in Norway exhibits an average of 3.73%. Put together, this implies that our estimate for the aggregate $R - G$ in Norway over the period considered amounts to 1.1%.

3.3 A micro-level perspective on *r* and *g*

A contribution of this paper is to present the first micro-level empirical estimates of the difference between the real rate of return and the growth rate of total fiscal pre-tax income, across the entire net wealth distribution. To this end, we define *r* as the percentile average (for each $p = 1, \ldots P$) of the individual rates of return:

$$r_{p,t} = \frac{1}{N_p} \sum_{i=1}^{N_p} \frac{k_{i,t}}{gw_{i,t-1}}, \tag{4}$$

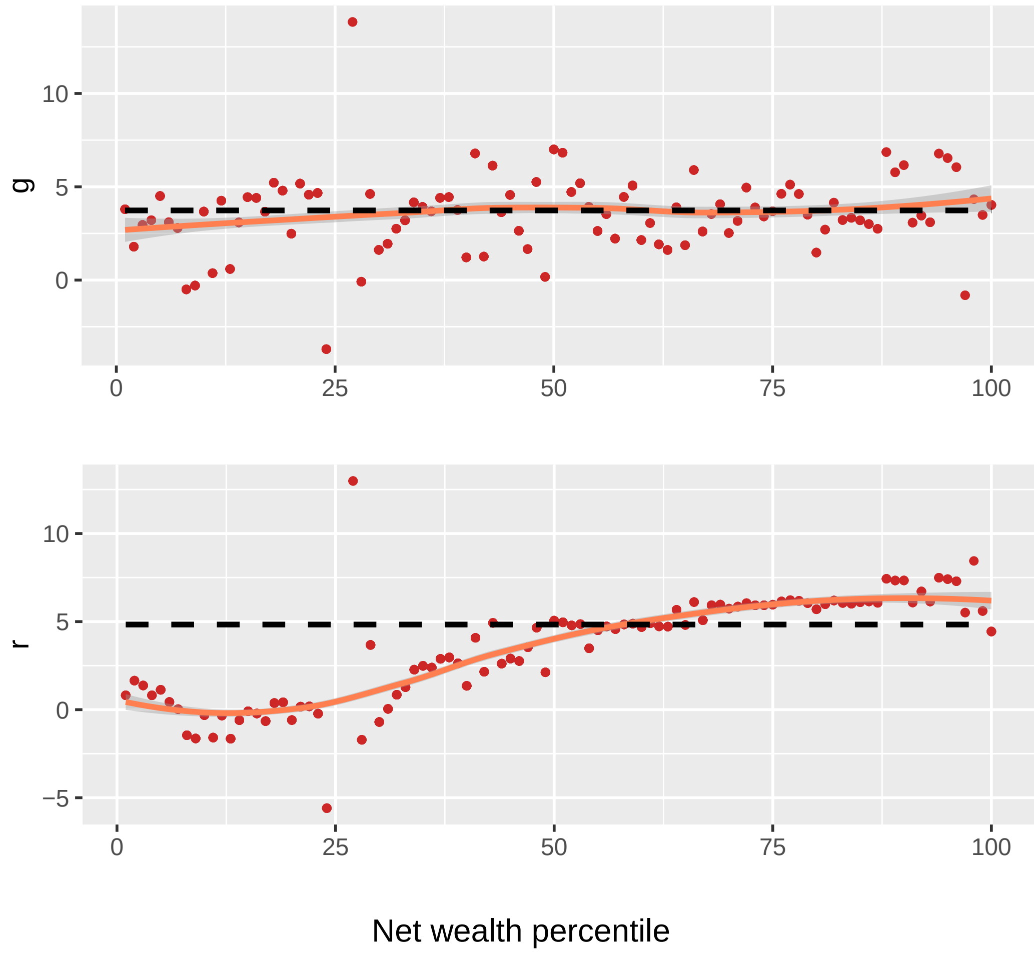
with N_p being the total amount of individuals in each percentile p. The standard deviation of the micro r_p is approximately 28%, slightly higher than the standard deviation of 22.1% estimated for unweighted returns to wealth in Fagereng et al. (2020) (although their analysis is based on the years $2004 - 2015$, hence it overlaps with our empirical exercise only for a few years). Regarding *g*, we define it as follows:

$$g_{p,t+1} = \frac{y_{p,t+1} - y_{p,t}}{y_{p,t}} = \frac{\sum_{i=1}^{N_p} y_{i,t+1} - \sum_{i=1}^{N_p} y_{i,t}}{\sum_{i=1}^{N_p} y_{i,t}}, \tag{5}$$

with N_p being the total amount of individuals in each percentile p of the net wealth distribution. At this point, we trim the full sample by excluding percentiles of r and g lying outside the accepted range of $[-30\%; +30\%]$. Volatility in the rates of return across the net wealth distribution is especially high for percentiles exhibiting low levels of gross wealth, whilst high volatility in growth rates of income is mostly due to capital gains on housing wealth. These corrections are conservative and, if anything, they reduce the extent of heterogeneity of r and g across the net wealth distribution.

Figure 6 shows how r and g evolve across the net wealth distribution, pooled across the years $(2012-2018)$. The dashed horizontal lines in the upper and lower parts of Figure 6 represent the levels for G and R, respectively.

Figure 6: The distribution of g and r



Note: this figure shows how g and r evolve across the net wealth distribution over the period considered. The dashed horizontal lines represent the aggregate levels for G and R, respectively. A local polynomial non-parametric fit for each of the two distributions is drawn.

In the upper part of Figure 6, it is shown that the micro g fluctuates around its aggregate counterpart G for the whole distribution of net wealth. On the contrary, the scatter in the lower part of Figure 6 shows that r exhibits higher heterogeneity and a positive degree of covariation with position in the net wealth distribution. The extent to which the covariation between r and net wealth holdings is due to heterogeneity or scale effects (or both) shall be further analyzed in the discussion section of the paper.

Before we move on to present the result for the micro $r - g$, let us highlight for the sake of clarity the analytical expression linking the macro $R - G$ to its micro counterpart. Recall the definition of the aggregate R in Equation 2, which can be expressed as a function of the micro r_p as follows:

$$R_t(r_{p,t}) = \frac{1}{P}\sum_{p=1}^{P}\left(\frac{1}{N_p}\sum_{i=1}^{N_p}\frac{k_{i,t}}{gw_{i,t-1}}\right) = r_{1,t}S_1 + \ldots + r_{p,t}S_p, \tag{6}$$

with $S_p = w_p/W$ being the share of net wealth within percentile p (hence $\sum_{p=1}^{P} S_p = 1$). In other words, the aggregate rate of return R can be decomposed into a weighted average of the micro rates at the percentile level. A similar decomposition can be applied to the growth rate of total income G of Equation 3, yielding the following result for the functional form of the difference between the macro $R - G$ and its micro counterpart:

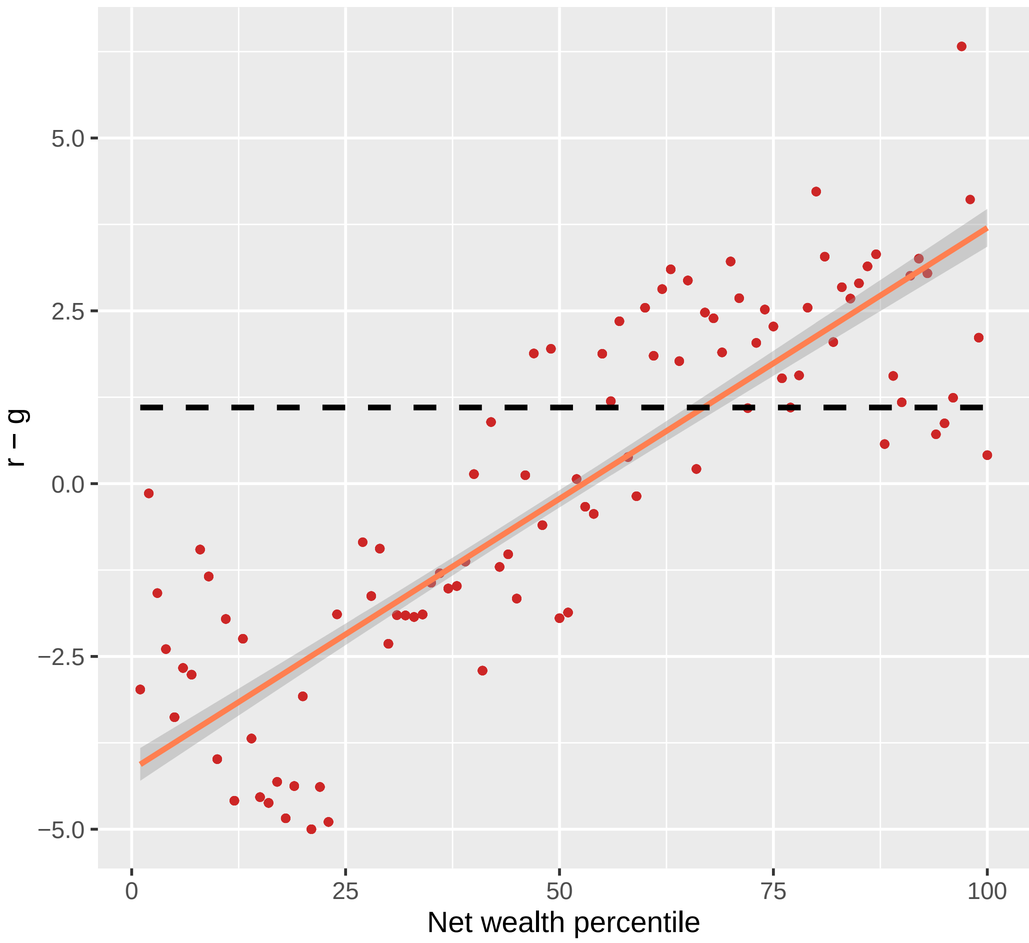
$$\begin{aligned} R_t(r_{p,t}) - G_t(g_{p,t}) &= (r_{1,t}S_1 + \ldots + r_{p,t}S_p) - (g_{1,t}\lambda_1 + \ldots + g_{p,t}\lambda_p), \\ &= (r_{1,t}S_1 - g_{1,t}\lambda_1) + \ldots + (r_{p,t}S_p - g_{p,t}\lambda_p). \end{aligned} \tag{7}$$

with $\lambda_p = y_p/Y$ being the share of total income within percentile p (hence $\sum_{p=1}^{P} \lambda_p = 1$).

3.4 The distribution of $r - g$

We present now the main finding from our empirical analysis, namely, the distribution of $r - g$. Figure 7 shows the difference between the *micro* rates of return on wealth r and the *micro* growth rate of personal total fiscal income g, across the net wealth distribution. The horizontal dashed line represents the aggregate $R - G$ with an average of 1.1% throughout the period, as shown in sub-section 3.2.

Figure 7: The distribution of $r - g$



Note: this figure shows the difference between the rate of return r and the growth rate of personal total fiscal income g, across the net wealth distribution, in percentage terms (%) and pooled across the years $2012 - 2018$. The horizontal dashed line represents the aggregate $R - G$ with an average of 1.1% throughout the period. A linear fit is drawn for illustrative purposes throughout the distribution of $r - g$.

The aggregate $R - G$ of 1.1% overestimates its micro counterpart $r - g$ for the bottom half (approximately for the bottom 60% of the net wealth distribution), whilst the opposite happens for the top 40%. We claim that this evidence demonstrates that an assessment of how the difference between the real rate of return on wealth minus the real growth of income is distributed, delivers additional insights than just focusing on mean variables. Therefore, the micro $r - g$ qualifies as a more precise measure to highlight distributional concerns.

The next sub-section illustrates, through a simple simulation exercise, a comparison between the adoption of the micro $r - g$ or the aggregate $R - G$ in order to understand the evolution of income and wealth inequality.

3.5 A simulation exercise

Does the micro $r-g$ predict higher or lower inequality of income and wealth with respect to its aggregate counterpart $R-G$? This subsection sheds light on this aspect, related to the dynamics of wealth inequality. Berman, Ben-Jacob, and Shapira (2016) study the dynamics of wealth inequality through a theoretical exercise based on a realistic modeling of the wealth distribution. In subsequent related study, Berman and Shapira (2017) analyze the asymptotic properties of the wealth distribution, concluding that for $r > g$ the wealth distribution constantly becomes more and more inegalitarian.

In the following, we wish to highlight the main finding of the empirical analysis of this paper, in a synthetic manner. We carry out a simulation calibrated on our data and we draw a comparison between two benchmark scenarios. Assume that wealth accumulation over periods can be summarized as follows $w_{p,t} = (1 + r_{p,t})w_{p,t-1}$, hence abstracting from simplicity from the additive role of savings, gifts and inheritances. Assume as well a fixed rank for both income and wealth distributions.

Now, let us draw two scenarios. In scenario A, we let each percentile[7] of the income distribution y_p grow for ten periods at the average growth rate estimated ($G = 3,73\%$), and we let each percentile of the wealth distribution w_p grow by a rate of return equal to the aggregate rate of return ($R = 4,83\%$). In a nutshell, scenario A depicts a situation in which $R-G = 1,1\%$ is constant across the wealth distribution, as shown by the horizontal line drawn in Figure 7. In scenario B, we introduce heterogeneity by allowing the percentiles of the income distribution to grow at the percentile level income growth rates (i.e., $g = g_p$), and we apply the micro rates of return ($r = r_p$) to the percentiles of the wealth distribution.

Table 1: Simulating income and wealth inequality dynamics

		t = 10	
	t = 1	Scenario **A**	Scenario **B**
Gini Income	0.27	0.27	0.29
(% change)		(0)	(7%)
Gini Net Wealth	0.61	0.61	0.62
(% change)		(0)	(2%)
Gini Gross Wealth	0.54	0.54	0.59
(% change)		(0)	(9%)

Note: scenario A applies average growth rates to all percentiles (G and R), scenario B applies percentile-specific growth rates ($g = g_p$ and $r = r_p$). Gini coefficients across net wealth percentiles are calculated at time $t = 1$ and time $t = 10$ for income, net wealth and gross wealth. % changes in parenthesis.

Results of the simulation exercise are presented in Table 1. The univariate Gini coefficients for income and wealth do not change after ten time periods under the scenario A (the $R-G$ scenario). On the other hand, introducing heterogeneity by allowing percentiles

[7] Each percentile is initialized with the average percentile-specific value over the period $2010-2018$ for the different variables considered in this exercise.

of the income and wealth distribution to grow at different rates in scenario B, it delivers a different outcome. The Gini coefficient of income increases by 7%, whilst the Gini of wealth increases by 2% considering net wealth or by 9% for gross wealth.

In other words, the aggregate $R-G$ implies an underestimation of the increase in both income and wealth inequality when information on how $r-g$ is distributed is not available. In our view, this result is in line with the theoretical insights in Stiglitz (2016), extending the Solow model by introducing variable returns to capital in order to explain the emergence of income and wealth inequality, and with Gabaix, Lasry, Lions, and Moll (2016) studying the importance of scale dependence in growth dynamics for understanding inequality. Piketty (2015b) clarifies how $R > G$ does not work as a direct determinant of inequality but instead as an amplifier of other kinds of shocks, increasing inequality in steady-state and making disparities more persistent. Our findings suggest that the heterogeneity of $r-g$ across the distribution should be added to the list of determinants of increasing economic inequality.

4 Discussion

4.1 Persistent heterogeneity or scale dependence?

To what extent is the main finding shown in Figure 7 caused by persistent heterogeneity across the net wealth distribution, and to what extent is it instead caused by scale effects? By persistent heterogeneity, we mean idiosyncracies in returns, which may for instance be attributed to differences in risk preferences or ability to catch entrepreneurial opportunities. A high degree of persistent heterogeneity implies that the aggregate $R-G$ fails to predict each and any single realization of the micro $r-g$, however, no clear covariation is visible between the micro $r-g$ and position in the net wealth distribution. By scale dependence, we mean a positive effect of the scale of net wealth on returns. If scale dependence is also causing variation in $r-g$, then we might observe an increasing monotonic trend in $r-g$, as it is indeed the case in Figure 7.

The implications of the above question are decisive for the study of wealth inequality. As argued by Piketty (2014), *"It is perfectly possible that wealthier people obtain higher average returns than less wealthy people... It is easy to see that such a mechanism can automatically lead to a radical divergence in the distribution of capital."* To investigate the relative importance of scale effects, we follow both Fagereng et al. (2020) and Gabaix et al. (2016) and estimate the following simple model:

$$(r-g)_{p,t} = \theta D_t + \omega_p + f_t + \epsilon_{p,t}, \tag{8}$$

where $(r-g)_{p,t}$ denotes the micro $r-g$ for percentile p at time t, D_t represents the decile of the net wealth distribution (capturing scale effects), ω_p and f_t are the percentile (capturing persistent heterogeneity) and time fixed effects (capturing time-dependent co-variation in $r-g$ and net wealth), respectively, and $\epsilon_{p,t}$ is the error term. In other words, the coefficient θ represents the scale dependence parameter. Since no other controls are included, this parameter includes direct and indirect scale dependence effects. Table 2 shows the results.

Table 2: Explaining heterogeneity in the distribution of $r - g$

	(1)	(2)	(3)
	$r - g$	$r - g$	$r - g$
Decile	0.777***	0.461***	0.461***
	(30.23)	(21.15)	(20.70)
Time FE	NO	NO	YES
Percentile FE	NO	YES	YES
Observations	487	487	487

t statistics in parentheses

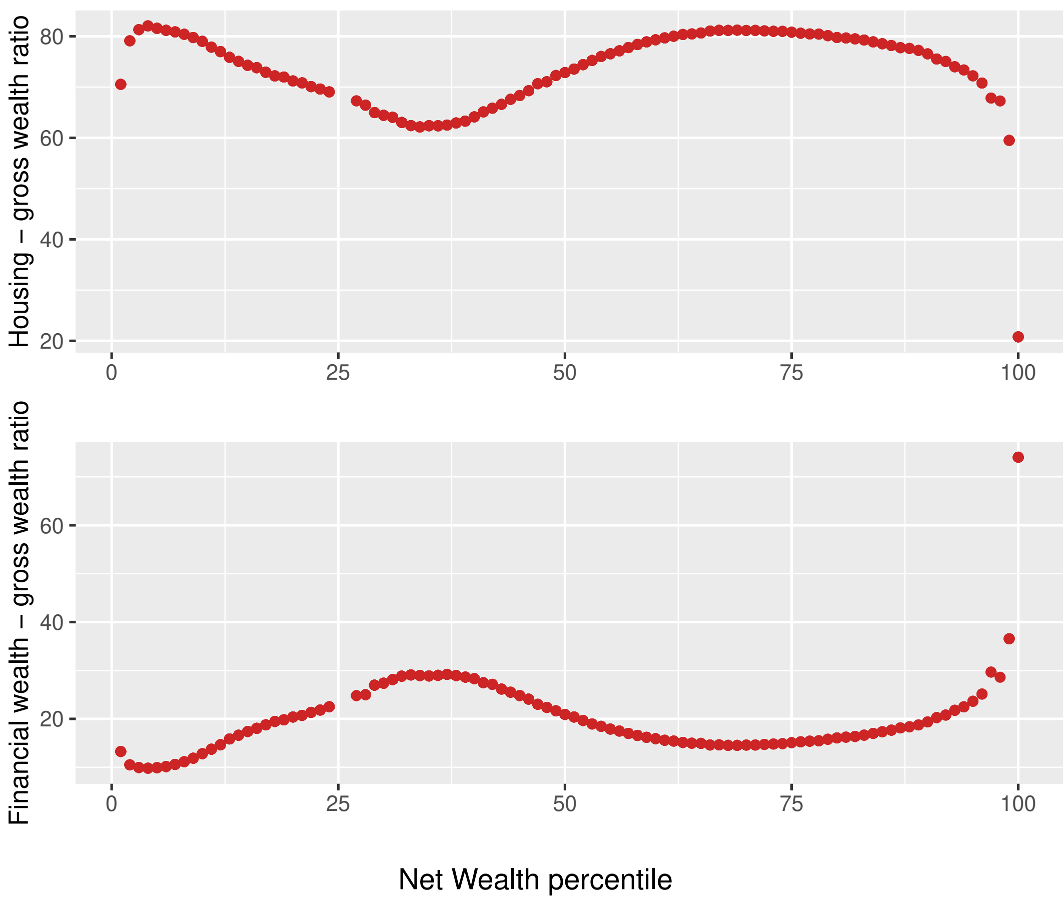
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Let us make sense of these results. The $P90/P10$ ratio for the micro $r - g$ plotted in Figure 7 is of 8.377 percentage points, indicating that by moving from the bottom to the top decile of the net wealth distribution (hence jumping up 8 deciles in total), the $r - g$ increases correspondingly by this magnitude. How much of this increase is attributable to scale dependence? According to the scale dependence coefficient θ in column 3 (when persistent heterogeneity is controlled for) each decile shift leads to an increase in $r - g$ of 0.461%, implying that jumping from the lowest to the highest decile corresponds to a higher $r - g$ of 3.688% (8 times θ). This amounts to approximately 44% of the whole change in $r - g$, indicating that scale dependence matters almost as much as persistent heterogeneity in explaining our main evidence. The coexistence of persistent heterogeneity and scale dependence results as well from the analysis in Fagereng et al. (2020) (although their focus is solely on the rates of return and not on $r - g$).

4.2 Do types of wealth and rates of return correlate?

This subsection focuses on a decomposition of our wealth series. Not all wealth owners are equal, and type of wealth substantially impacts rates of return. Focusing now only on r, do we gain additional insights by separating between type of wealth owners? We compute for all years the shares of housing (including estimated market value of first and secondary dwellings) and financial wealth on personal gross wealth, with individuals ranked by their position in the net wealth distribution, and we plot it in Figure 8. As shown, housing represents the main wealth component for the middle class $50 - 90\%$, since it stands for around $75 - 80\%$ of their gross wealth. Focusing on the top 10%, the picture changes slightly. Housing remains the biggest component of gross wealth for the $90 - 99$ percentiles although with a lower share, before it drops to around 20% of total gross wealth for the top 1%.

Figure 8: Financial wealth and housing shares of gross wealth



Note: this figure shows the shares of housing (including estimated market value of first and secondary dwellings) and financial wealth on personal gross wealth, with individuals ranked by their position in the net wealth distribution.

We specify a baseline linear fixed-effects model to synthesize the information on types of wealth and rates of return:

$$r_{p,t} = \omega_p + f_t + \rho_{p,t} + \mu_{p,t} + \gamma X_{p,t} + \epsilon_{p,t}, \tag{9}$$

where $r_{p,t}$ denotes the rate of return r for percentile p at time t. ω_p and f_t are the fixed percentile and time effects, respectively. $\rho_{p,t}$ is the lagged share of financial wealth on gross wealth for each percentile, whilst $\mu_{p,t}$ is the lagged share of housing. $X_{p,t}$ represents a set of control variables (lagged levels of housing and financial wealth), and $\epsilon_{p,t}$ is the error term.

Results are shown in Table 3. In model specifications $[1-4]$, the lagged share of financial wealth is included as the main regressor, in addition to control variables such as percentile and time fixed effects and lagged levels of financial wealth. A 1% increase in the lagged share of financial wealth owned within the percentile, leads to a 2.516% increase in r (column 4), implying that the type of wealth matters, and that an increasing share of financial wealth leads to higher returns for large owners of financial wealth. In model

Table 3: Explaining rates of return in relation to type of wealth owners

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	r	r	r	r	r	r	r	r
Financial (%)	0.249	1.654*	1.907*	2.516**				
	(0.40)	(2.17)	(2.05)	(3.13)				
Housing (%)					-1.777***	-1.180	-1.215	-1.912**
					(-3.47)	(-1.81)	(-1.70)	(-3.08)
Percentile FE	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	NO	YES	YES	YES	NO	YES	YES	YES
Housing (level)	NO	NO	NO	NO	NO	NO	YES	YES
Financial (level)	NO	NO	YES	YES	NO	NO	NO	NO
Time x Level	NO	NO	NO	YES	NO	NO	NO	YES
Observations	291	291	291	291	291	291	291	291

Note: The table shows regression estimates of the micro r as in model specification given by equation 9. All regressions include a full set of dummies for net wealth percentiles computed on 1-year lagged housing wealth (both in levels and as a share of gross wealth), financial wealth (both in levels and as a share of gross wealth). YES implies that the regressor is included, NO that it is not. t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

specifications $[5-8]$, we include the lagged share of housing wealth as the main regressor, in addition to control variables such as percentile and time fixed effects and lagged levels of housing wealth. As expected, a 1% higher share of housing wealth is expected to lead to a 1.912% lower r (column 8), contrary to what we observed for financial wealth.

5 Concluding remarks

This paper analyzes for which fractions of the net wealth distribution returns to wealth happen to be higher than growth rates of income, by utilizing Norwegian administrative tax records on income and wealth. The implication of this analysis for the study of the dynamics of income and wealth inequality is that an entire distribution of $r-g$, by allowing for covariation between real rates of return and position in the net wealth distribution, qualifies as a more precise measure than simply focusing on the aggregate $R-G$.

Our main contribution is to show that, for the top half of the distribution, the aggregate $R-G$ underestimates its micro counterpart $r-g$, whilst the opposite is true for the bottom half. We investigate the determinants of this variability and show that around 44% of the variation in $r-g$ when moving up from the bottom to the top decile of the net wealth distribution is associated with scale dependence, whilst the rest is due to persistent heterogeneity.

In our view, this empirical exercise confirms the relevance of taking into account substantial heterogeneity when modeling inequality in relation to macroeconomic phenomena. We also believe that this study enhances our understanding of the relevance of the measure $r-g$ for the study of inequality, although it leaves aside important aspects as the role of public wealth and retained earnings. If anything, we presume that allocating undistributed profits would imply even stronger heterogeneity of $r - g$ across the wealth distribution, hence reinforcing the main message of this work.

References

Alvaredo, F., Atkinson, A. B., Chancel, L., Piketty, T., Saez, E., & Zucman, G. (2016). Distributional National Accounts (DINA) Guidelines: Concepts and Methods used in WID.world. *WID Working Paper 2016 No.2*.

Bach, L., Calvet, L. E., & Sodini, P. (2020). Rich Pickings? Risk, Return, and Skill in Household Wealth. *American Economic Review*, *110*(9), 2703-47. Retrieved from `https://www.aeaweb.org/articles?id=10.1257/aer.20170666`

Berman, Y., Ben-Jacob, E., & Shapira, Y. (2016). The dynamics of wealth inequality and the effect of income distribution. *PLOS ONE*, *11*(4), 1-19. Retrieved from `https://doi.org/10.1371/journal.pone.0154196`

Berman, Y., & Shapira, Y. (2017). Revisiting $r > g$ — The asymptotic dynamics of wealth inequality. *Physica A: Statistical Mechanics and its Applications*, *467*, 562 - 572. Retrieved from `http://www.sciencedirect.com/science/article/pii/S0378437116307221`

Bø, E. E. (2020). Taxation of housing: Killing several birds with one stone. *Review of Income and Wealth*, *66*(3), 534-557. Retrieved from `https://onlinelibrary.wiley.com/doi/abs/10.1111/roiw.12423`

Epland, J., & Kirkeberg, M. (2012). *Wealth Distribution in Norway: Evidence from a New Register-Based Data Source* (Reports No. 35). Statistics Norway. Retrieved from `https://www.ssb.no/a/english/publikasjoner/pdf/rapp_201235_en/rapp_201235_en.pdf`

Fagereng, A., Guiso, L., Malacrino, D., & Pistaferri, L. (2020). Heterogeneity and persistence in returns to wealth. *Econometrica*, *88*(1), 115-170. Retrieved from `https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA14835`

Fagereng, A., Holm, M. B., Moll, B., & Natvik, G. (2019). *Saving behavior across the wealth distribution: The importance of capital gains* (Working Paper No. 26588). National Bureau of Economic Research. Retrieved from `http://www.nber.org/papers/w26588` doi: 10.3386/w26588

Gabaix, X., Lasry, J.-M., Lions, P.-L., & Moll, B. (2016). The Dynamics of Inequality. *Econometrica*, *84*(6), 2071-2111. Retrieved from `https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA13569`

Hiraguchi, R. (2019). Wealth inequality, or r – g, in the economic growth model. *Macroeconomic Dynamics*, *23*(2), 479–488. doi: 10.1017/S1365100516001206.

Jones, C. I. (2015). Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality. *Journal of Economic Perspectives*, *29*(1), 29-46. Retrieved from `https://www.aeaweb.org/articles?id=10.1257/jep.29.1.29`

Jordà, O., Knoll, K., Kuvshinov, D., Schularick, M., & Taylor, A. (2019). The Rate of Return on Everything, 1870–2015. *The Quarterly Journal of Economics*, *134*(3), 1225-1298. Retrieved from `https://doi.org/10.1093/qje/qjz012`

Mankiw, N. G. (2015). Yes, $r > g$. So What? *American Economic Review*, *105*(5), 43-47. Retrieved from `https://www.aeaweb.org/articles?id=10.1257/aer.p20151059`

Piketty, T. (2014). *Capital in the Twenty-First Century*. Harvard University Press. Retrieved from `http://www.jstor.org/stable/j.ctt6wpqbc`

Piketty, T. (2015a). About Capital in the Twenty-First Century. *American Economic Review*, *105*(5), 48-53. Retrieved from `https://www.aeaweb.org/articles?id=10.1257/aer.p20151060`

Piketty, T. (2015b). Putting distribution back at the center of economics: Reflections on capital in the twenty-first century. *Journal of Economic Perspectives*, *29*(1), 67–88.

Piketty, T., & Zucman, G. (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *The Quarterly Journal of Economics*, *129*(3), 1255-1310. Retrieved from `https://doi.org/10.1093/qje/qju018`

Stiglitz, J. E. (2016). New Theoretical Perspectives on the Distribution of Income and Wealth Among Individuals. In *Inequality and growth: Patterns and policy.* doi: 10.1057/9781137554543_1

Appendix A: Descriptive statistics

Table 4: Descriptive statistics of the baseline sample.

Variables	Categories	Unit	Obs	Mean	St. Dev.	Min	Max
Personal Gross Wealth - (gw)	2010	NOK	3 672 154	1463380.1028	1685698.0736	0	9329716
	2011	NOK	3 736 564	1554668.584	1788062.487	0	9829390
	2012	NOK	3 796 370	1683743.0032	1907787.5646	0	10462519
	2013	NOK	3 850 558	1757966.1676	2011619.0821	0	11067254
	2014	NOK	3 904 534	1814654.1231	2066818.6761	0	11455698.33
	2015	NOK	3 951 412	1942130.2156	2217501.5086	0	12424534
	2016	NOK	3 995 151	2079815.851	2399362.3057	0	13564935
	2017	NOK	4 061 631	2206908.234	2592048.4519	0	14868726.56
	2018	NOK	4 126 767	2234013.351	2641582.311	0	15340798
	2010-2018		35 095 141				
Private Debt - (d)	2010	NOK	3 672 154	572006.9409	887600.6338	0	4796267
	2011	NOK	3 736 564	604197.2627	938542.5002	0	5038880
	2012	NOK	3 796 370	640141.8714	992279.3503	0	5301895
	2013	NOK	3 850 558	675494.5575	1046485.1423	0	5601334
	2014	NOK	3 904 534	705418.2386	1090841.0443	0	5817836
	2015	NOK	3 951 412	738784.8513	1143644.2329	0	6070201
	2016	NOK	3 995 151	774263.3705	1201549.8505	0	6367489
	2017	NOK	4 061 631	807733.4229	1260241.406	0	6684241
	2018	NOK	4 126 767	837090.6793	1309206.2736	0	6941879
	2010-2018		35 095 141				
Personal Net Wealth - (w)	2010	NOK	3 672 154	891373.1619	1581126.2314	-4796267	9329716
	2011	NOK	3 736 564	950471.3213	1670687.3286	-5038880	9829390
	2012	NOK	3 796 370	1043601.1318	1768568.4121	-5301895	10462519
	2013	NOK	3 850 558	1082471.61	1871589.4863	-5601334	11067254
	2014	NOK	3 904 534	1109235.8845	1926933.7456	-5817836	11455698.33
	2015	NOK	3 951 412	1203345.3643	2065344.3123	-6070201	12424534
	2016	NOK	3 995 151	1305515.2237	2231434.3546	-6367489	13564935
	2017	NOK	4 061 631	1399136.5817	2402849.0841	-6684241	14868726.56
	2018	NOK	4 126 767	1396883.9111	2452349.2445	-6941879	15340798
	2010-2018		35 095 141				
Capital income - (k)	2010	NOK	3 672 154	10388.7938	36130.7456	-6255	287378
	2011	NOK	3 736 564	11671.6094	39172.2868	-12011	307958
	2012	NOK	3 796 370	12574.3758	41266.2958	-3030	324819
	2013	NOK	3 850 558	14212.711	46434.3561	0	364900
	2014	NOK	3 904 534	15133.9046	50644.2364	-4918	400050
	2015	NOK	3 951 412	15467.0957	62335.6454	-14240	510809
	2016	NOK	3 995 151	12690.2313	54444.6483	-16499	446348
	2017	NOK	4 061 631	13521.0656	57585.0475	-16063	473962
	2018	NOK	4 126 767	13531.2799	56577.9826	-6494	468565
	2010-2018		35 095 141				
Total fiscal income - (y)	2010	NOK	3 672 154	370486.8579	241542.426	0	1441504
	2011	NOK	3 736 564	387346.7956	255816.7936	0	1525017
	2012	NOK	3 796 370	403894.1623	268705.8449	0	1593909
	2013	NOK	3 850 558	419618.4645	281579.7698	0	1665686
	2014	NOK	3 904 534	432801.0258	292761.3459	0	1737556
	2015	NOK	3 951 412	445533.6917	307390.4366	0	1888036
	2016	NOK	3 995 151	449121.896	302836.0827	0	1821027
	2017	NOK	4 061 631	454857.311	311373.0565	0	1859382
	2018	NOK	4 126 767	463823.6129	323328.2209	0	1913067
	2010-2018		35 095 141				

Note: This Table presents the summary statistics of our baseline sample. Our sample is constructed by taking into account the entire population of residents with age 20 years and above, in between $2010 - 2018$. All variables are pre-tax and are considered at the last day of the year.